A phase-image-based content-addressable holographic storage with security

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ABSTRACT
We demonstrate content-addressable holographic data storage with security using random phase encoding in the Fresnel domain. The phase-based data pages are encrypted using a random phase mask in the Fresnel domain, and recorded on a photorefractive LiNbO$_3$: Fe crystal using angular multiplexing. While content-searching through this database, the correct random phase mask and its position are critical to perform the search. Hence only an authentic user who has got the full key information can perform search through the database. Apart from the security, the use of random mask is most important in ensuring the fidelity of the search. The random mask ensures that the correlation peak intensity obtained is proportional to the inner product between the stored data page and the search argument. We have also studied the search capability by using partial search arguments of varying sizes. The lateral and the longitudinal shift sensitivities of the random phase mask while content searching have been studied. It has been found that an in-plane shift of 150 µm or an out-of-plane shift of 200 µm reduces the correlation peak intensity drastically so that no practical search is possible. We have carried out investigations on discrimination capabilities of the system, using the parameters discrimination ratio and SNR for various sizes of the search argument.

Keywords: Holographic data storage, associative retrieval, data security, random phase encoding

1. INTRODUCTION
Holographic data storage systems, due to their high capacity, parallel access, fast data transfer rates, and fast data search capabilities, are potential candidates for storing large volumes of data. Fast parallel search is possible using the principle of associative retrieval in a holographically multiplexed databank. In addition, optical encryption techniques allow securing the optically stored data by preventing access to unauthenticated users. There are a number of ways of securing data in a holographic memory system. One of the most successful and secure methods for optical encryption is the random phase encoding method. Many researchers have demonstrated a secure memory using optical encryption in the Fourier domain and Fresnel or fractional Fourier domains, using single random and double random phase encoding. One important application of optical encryption techniques is in developing a secure optical database in which only authentic users having the correct key information can search through the database as well as retrieve the dataset.

Most of the current technologies for querying through a database are time-consuming and not suitable for real-time application. The property of content-addressability can be exploited for fast content search operations in a holographic data storage system. Earlier works in this area have clearly demonstrated the strong potential and wide applications of this technique in a practical binary database environment, which cannot be provided by the current data storage technologies that are serially addressed. In the present paper, we demonstrate content-addressability in an encrypted phase image-based holographic data storage system. We have carried out parallel searches in a holographic memory encrypted using a random phase mask in the Fresnel domain. The correct phase mask as well as its three-dimensional position is critical in carrying out a search in the database. We have also carried out searches using partial search arguments to see how small the search argument could be in order to carry out the search for an authentic user having full information about the keys.

2. PRINCIPLE
Content addressability is one of the unique properties of a volume holographic memory which allows simultaneous search of an entire database in a single step by performing multiple optical correlations between stored page and the

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search argument. When a holographically multiplexed data bank is readout with a search argument, the search argument gets multiplied with all the stored pages of the databank and reconstructs a set of reference beams used to multiplex each stored page. The intensity of each reference beam is proportional to the inner product between the search and the stored data page. Though various researchers have demonstrated the potential of content addressing in a binary database environment, a few drawbacks have been highlighted which needs to be addressed in order to present a foolproof system. Fidelity of search is one issue of serious concern. Betzos et al. have proposed a method for reducing the number of false hits while performing search.

We have proposed the use of phase-based data pages and use of hybrid ternary modulations for improved performance in content addressable systems. The main aim of this paper is to demonstrate a secure system using random phase encoding in which only an authentic user can perform search through the database. Use of a random phase mask has two main significances.

1. A random phase mask encrypts the data recorded. Hence only an authentic user who has full information about the key as well as its 3-D position only can search the data as well as retrieve it.

2. A random phase mask with specific statistical properties ensures that the intensity of the correlation peaks is proportional to the inner product between the stored page and the search argument while performing content addressing.

It has to be noted that in a holographic storage system with a generalized geometry where recording plane is not necessarily the Fourier plane, the use of a random phase mask which is a stationary white noise is essential for to perform reliable content-based search.

Grawert et al. have discussed in detail, the ambiguous behavior of the correlation scores while performing content addressing in a defocused correlator geometry. They have proposed the use of a random phase mask as a solution to this problem. In this paper, we model a phase-based content-addressable holographic system with data encrypted in the Fresnel domain using a random phase mask. Using the basic equations for Fresnel propagation, we show that while content addressing in such an encrypted database, the intensity of the diffracted reference beams will be proportional to the inner product between the search argument and the stored pages. We experimentally demonstrate the system using a database of 16 angularly multiplexed data pages. We also study the shift sensitivities of the random phase mask while performing content addressing. We perform simulation studies on the discrimination capabilities of the system, using discrimination ratio and SNR and compare results with the experimental results.

The optical set-up for image encryption and content addressing is represented by the schematic shown in fig. 1. For convenience of analysis, we take the functional representation in one dimension which can easily be extended to 2 dimensions.

Let the input function be denoted by \( f(x_1) \). After propagating through a distance \( d_1 \), the function can be represented as

\[
f_1(x_2) = \int f(x_1) \exp \left[ \frac{-i\pi}{x d_1} (x_2 - x_1)^2 \right] dx_1
\]

(1)

The lens with a focal length \( f \) introduces a quadratic phase factor and the resultant function after the lens can be represented as

\[
f_2(x_2) = f_1(x_2) \exp \left[ -i\pi x_2^2 \right]
\]

(2)

This after free-space propagation through a distance \( d_2 \) can be represented as
The resultant function after getting multiplied with the random phase mask can be represented as $f_3(x_3) R(x_3)$. After multiplication the resultant function propagates through a distance $d_3$ where it is recorded in a photorefractive crystal. The value of the function at the recording plane can be written as

$$f_4(x_4) = \int f_3(x_3) R(x_3) \exp\left[\frac{i\pi}{\lambda d_3} (x_4 - x_3)^2\right] dx_3$$

Substituting the value of $f_3(x_3)$ given by (5) into (4), $f_4(x_4)$ can be expressed as

$$f_4(x_4) = \int f(x_1) \exp\left[\frac{i\pi}{\lambda d_1} (k_1 x_1^2 - 2k_2 x_1 x_3 + k_3 x_3^2)\right] dx_1$$

where

$$k_1 = \frac{\int d_1 x_2^2 - d_1 x_2 d f - \int d_1 x_2}{\int d_1 x_2^2 - d_1 x_2 d f}$$

$$k_2 = \frac{\int d_2 x_2^2 - d_2 x_2 d f}{\int d_2 x_2^2 - d_2 x_2 d f}$$

$$k_3 = \frac{\int d_3 x_2^2 - d_3 x_2 d f}{\int d_3 x_2^2 - d_3 x_2 d f}$$

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This wave is allowed to interfere with a plane reference beam $S(x)$ leading to hologram formation which can be represented as

$$\left|S(x) + \tilde{f}_4(x)\right|^2 = \left|S(x)\right|^2 + \left|\tilde{f}_4(x)\right|^2 + S(x) \tilde{f}_4^*(x) + S^*(x) \tilde{f}_4(x)$$

where the symbol $\tilde{}$ denotes the function which is stored in the form of hologram.

The term contributing to the correlation signal while performing a content-based read-out is the 3rd term in (7).
\[ E[f_4(x_4)S(x_4)f_4^*(x_4)] = S(x_4)\left[ \frac{\partial}{\partial x_3} f(x_1) \right] \]
\[ \times \exp\left[ \frac{i\pi}{\lambda} \left( \frac{k_3 x_3^2}{d_1} - \frac{2k_2 x_1 x_3}{d_1} - \frac{2x_3 x_4}{d_1} \right) \right] \exp\left[ \frac{i\pi}{\lambda} \left( \frac{k_3 x_3^2}{d_1} - \frac{2k_2 x_1 x_3}{d_1} - \frac{2x_3 x_4}{d_1} \right) \right] \]
\[ \times E[R(x_3)R(x_3)]dx_1 dx_3 dx_3 dx_3 \]  

Since \( R(x_3) \) is a stationary white noise,
\[ E[R(x_3)R(x_3)] = \delta(x_3 - x_3) \]  

Substituting (9) into (8), we get
\[ E[f_4(x_4)S(x_4)f_4^*(x_4)] = S(x_4)\left[ \frac{\partial}{\partial x_3} f(x_1) \right] \]
\[ \times \exp\left[ -\frac{i\pi}{\lambda} \left( \frac{k_3 x_3^2}{d_1} - \frac{2k_2 x_1 x_3}{d_1} - \frac{2x_3 x_4}{d_1} \right) \right] \]
\[ \times E[R(x_3)R(x_3)]dx_1 dx_3 dx_3 dx_3 \]  

The second term in (10) reduces to \( \delta(x_1 - x_1) \). Hence we can write the RHS of (10) as
\[ S(x_4)\left[ \frac{\partial}{\partial x_3} f(x_1) \right] \exp\left[ -\frac{i\pi}{\lambda} \left( \frac{k_3 x_3^2}{d_1} - \frac{2k_2 x_1 x_3}{d_1} - \frac{2x_3 x_4}{d_1} \right) \right] \delta(x_1 - x_1) \]
\[ \times dx_1 dx_3 dx_3 dx_3 \]  

which reduces to
\[ E[f_4(x_4)S(x_4)f_4^*(x_4)] = S(x_4)\left[ \frac{\partial}{\partial x_3} f(x_1) \right] \]
\[ \times dx_1 \]  

when \( x_1 = x_1 \).

This is the inner product between the two functions, the read-out function \( f(x_1) \) and the stored function \( \tilde{f}(x_1) \). We can generalize (12) to two dimensions, giving the result
\[ E[f_4(x,y)S(x,y)f_4^*(x,y)] = S(x)\left[ \frac{\partial}{\partial y_1} f(x_1,y_1) \right] \]
\[ \times dy_1 dx_1 \]  

The integral in the RHS of (13) represents the inner product between the stored and the read-out functions, and is a scalar quantity. Hence we can say that the RHS of (13) represents the reference beam diffracted from the stored hologram whose intensity is proportional to the inner product between the stored data page and the search argument. This proves that while content-addressing, the correlation peak intensity is proportional to the inner product between the stored and the search pages, irrespective of the plane of recording if we use a random phase mask which is a stationary white noise.

We have simulated the above model on a Matlab platform. We generated 15 phase data pages of 128 x 128 pixels. The pages were balanced using 6:8 modulation code. In this coding scheme, each data word of 6 bits is mapped to an 8 bit
codeword such a way that all the code words will be having four ‘0’s and ‘1’s. In phase based pages, instead of ‘0’ and ‘1’, the number of ‘0’ and ‘π’ are made equal using the above modulation code. One of the main advantages of balancing phase-based data pages is the removal of high intensity dc from the Fourier spectra. The numerical computations of Fresnel transforms were carried out using the algorithms described in ref. 18. We studied the discrimination capability of this system for varying sizes of the search argument. We perform all possible cross-, and auto correlations among the 15 data pages and estimate the correlation peak intensity values. The results have been plotted using the metric discrimination ratio (DR). DR is calculated using the formula given below

\[
DR = \frac{\text{crosscorrelation peak intensity}}{\text{autocorrelation peak intensity}}
\]

(14)

Result of DR Vs. size of search argument is shown in fig. 2. The simulation studies on DR show high discrimination capability of the system.

3. EXPERIMENTS

We have performed encryption of 16 digital data pages in the Fresnel domain. The data pages were of size 192 x 192 pixels with each bit represented by a group of 4 x 4 pixels. The data pages were displayed on an SLM (make Jenoptik) of size 624 x 832 pixels and pixel pitch 32 x 32 μm. The encrypted pages were angularly multiplexed on a photorefractive LiNbO3:Fe crystal of size 20 x 10 x 10 mm with the beams incident on the 20 x 10 mm face. The mean angle between the object and the reference beams was ~23°. The angular spacing between subsequent reference beams while multiplexing was ~0.08°. The experimental set-up is shown in fig. 3.

We have used a modification of the Hybrid Ternary Modulation (HTM) scheme proposed by Jang et al to achieve 0-π phase modulations on the SLM. The details of this coding technique are given in ref. 15. For the sake of completeness, a brief description of this modulation scheme is given here. In conventional amplitude-based data pages, ‘0’ is represented by an OFF pixel state and ‘1’ by an ON state, whereas in phase-based pages, data bits of ‘0’ and ‘1’ are represented by 0 and π phase modulations of the pixel. In HTM, instead of two states, there are three states to represent two bits of data. There are two bright states with a phase difference of π to represent the ‘1’ bit. The third state which is a totally dark state is achieved by applying a proper intermediate voltage to the pixels. This state is used to represent a ‘0’ bit. The three pixel states used to represent binary data in HTM scheme are shown in fig. 4a. In the new modified HTM scheme which we follow (fig 4b), the two bright states of HTM with a phase difference of π are used to code the binary data of ‘0’ bit and ‘1’ bit.

![Figure 2: DR plot with size of search argument for simulation and experimental cases.](image)

![Figure 3: Experimental set-up for the encrypted content addressable memory. SFBE: spatial filter-beam expander assembly, Ms: mirrors, BS: beam splitter, HWP: half wave plate, SLM: spatial light modulator, PRC: photorefractive crystal, Mrot: Mirror mounted on precision rotation stage, CCD: charge coupled device.](image)
The third state, which is a totally dark state, is not used for coding data; instead, it is used to code the undesired regions of the SLM while performing a search operation with a partial search argument. An example of displaying a binary data word ‘01011001’ in amplitude-, phase-, hybrid ternary-, and modified hybrid ternary-, modulation schemes is depicted in fig. 4(c-f). A black box represents a pixel state with zero amplitude transmittance and zero phase. A white box represents a pixel state with full amplitude transmittance.

Figure 4 (a): Three states of HTM scheme. Figure 4 (b): Three states of the modified HTM scheme.

Figure 4 (c-f): Representation of the binary codeword ‘01011001’ in (c) amplitude mode; (d) phase mode; (e) HTM mode; (f) modified HTM mode. (Black boxes represent the zero amplitude state. White boxes represent states with maximum transmittance. Gray boxes represent the ‘ZERO’ state of the HTM when an intermediate voltage is applied to the pixel. These three states of HTM are achieved by properly orienting a quarter wave plate and analyzer at the output side of the SLM. The details of achieving the HTM are not shown in the experimental set-up to avoid complications.

A gray box represents the ‘ZERO’ state of the HTM when an intermediate voltage is applied to the pixel. These three states of HTM are achieved by properly orienting a quarter wave plate and analyzer at the output side of the SLM. The details of achieving the HTM are not shown in the experimental set-up to avoid complications.

Figure 5: Correlations obtained from a databank of 16 pages for (a) 3rd page (b) 10th page
The input data page on modulating the laser beam is propagated through free-space for a distance \(d_1\), where it gets focused by the lens of focal length 135 mm. Again after Fresnel propagation of a distance \(d_2\), the wavefront gets multiplied with the random phase mask \(R_1\). After this, the resultant beam is again propagated through a distance \(d_3\) where it is recorded in the photorefractive crystal. Typical distances of \(d_1\), \(d_2\) and \(d_3\) were 100, 80, and 50 mm respectively. The random phase mask which we used was fabricated using gelatin following standard procedure described in ref. 2.

After multiplexing, the content-addressing was performed using the same random phase mask. The results (Fig. 5a-b) show the correlation peak intensities for two different data pages while using the exact random phase mask. After this we have studied the shift sensitivities of the random mask used. The shift sensitivities of the random phase mask in the lateral and longitudinal directions are shown in fig.6a, b. We have obtained an in-plane shift tolerance of 150 \(\mu\)m and an out-of-plane shift tolerance of 200 \(\mu\)m beyond which the correlation peak intensity reduces drastically so that no practical search is possible. Hence the random phase mask also acts as the key for securing the data. The original random mask and its three-dimensional position are needed for performing a search through the database. There are different techniques for retrieving the original data from the phase information\(^\text{20}\). The retrieval of original amplitude data from the phase information is beyond the scope of this paper since our main aim is to demonstrate the content-addressability and study the fidelity of search in an encrypted data base.

Studies were also carried out on the discrimination capability of the encrypted memory system while content-addressing with small search arguments. We used DR and SNR as the parameters for studying this. DR has been defined in eqn. 14.

The evaluation of SNR gives a clearer picture of discrimination capability in the presence of noise. We define SNR as
\[ \text{SNR} = \frac{\mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \]  

(15)

where, \( \mu_1 \) and \( \mu_2 \) are respectively the means of the autocorrelation peak intensities and highest cross correlation peak intensities, and \( \sigma_1 \) and \( \sigma_2 \) are respectively the standard deviations of the autocorrelation peak intensities and highest cross correlation peak intensities. The graph showing DR Vs size of search argument is shown in fig. 2. It has been found that while searching with search arguments of size below 12% of the original data page, we were not able to detect the correlation signal intensities properly. Apart from this, the DR ratio showed high discrimination capability and the behavior was similar to that of the simulation result. Similarly the graph showing the variation of SNR with size of search argument is also shown in fig. 7. The SNR values are greater than 10 for search arguments of size more than 20%. For search sizes greater than 60% of the stored page, the SNR values show a steady increase proving the high consistency of the search results. The graph shows that the SNR is low when the search argument size goes below 15%. Hence the results of searches obtained using search arguments of lower sizes may lead to ambiguity. Low values of SNR and DR when the size of search argument is low can be attributed to various noise factors and the dark signal contributions to the correlation signal.

4. CONCLUSIONS

We have demonstrated a phase-image based content-addressable holographic storage system with security. Only an authentic user who knows the full key information and its 3-D position can search through the database. We have analysed the shift sensitivities of the random phase mask used. The use of a random phase mask which is a stationary white noise not only secures the data to be stored, but also ensures that the correlation peak intensity is proportional to the inner product between the search argument and the stored data pages. Our experimental verification on a databank of 16 multiplexed data pages proves the concept of security while content addressing. A detailed analysis needs to be carried out on the discrimination capability while using very small search arguments.

REFERENCES