Ultrasonic Texture Motion Analysis: Theory and Simulation

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Abstract—A theoretical model was previously developed to evaluate the relationship between the dynamics of ultrasonic speckle and its underlying tissue. The model is divided into an instrumental part represented by the point spread function (in the far field) of the ultrasonic apparatus and a moving tissue component described by a collection of scatterers. By computing the convolution of these terms and then the envelope, one obtains a simulated ultrasonic speckle pattern sequence which shows speckle motions closely linked to the tissue dynamics when small motion amplitudes are involved. In this paper, a theoretical study of the correlation between various linear transformations of the tissue and the corresponding ultrasonic speckle motions is performed, based on a 2-D extension of the envelope cross-correlation analysis of a narrow-band Gaussian noise. In the linear scan case, obviously, tissue translation generates an identical speckle translation. However, tissue/speckle motion correlation decreases with increasing rotation and/or biaxial deformation, lateral deformation (perpendicular to the beam propagation axis) being much less sensitive. With respect to the transducer frequency, the rotation and the axial deformation of the tissue show a better relationship with their respective speckle motion at lower frequencies while lateral deformation correlation is independent of the pulse frequency. With respect to beam (pulse) size parameters, tissue/speckle correlation decreases with rotation when a wide ultrasonic beam is used while the axial deformation correlation decreases with the axial duration of the pulse. This study sets the ground for the development of an ultrasonic strain gauge particularly useful for the assessment of biomechanical soft tissue and fluid flow properties based on speckle tracking.

I. INTRODUCTION

FROM AN observer point of view, soft tissue ultrasonic speckles are randomly located. Therefore, as a general rule, many studies on ultrasonic image texture analysis address a tissue characterization problem using a statistical approach, i.e., by trying to relate first- and second-order statistical properties of the tissue texture to the tissue structure [1]. However, small changes in tissue position are expected to lead to corresponding observable changes in the speckles. Thus, it makes sense to study speckle tracking as a means to infer tissue dynamics.

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In order to get a better understanding of the relationship between speckle and tissue position changes, we first developed a computer model of the B-scan imaging process associated with a contracting myocardium [2]. Using the model, we reported speckle tracking feasibility (and limitations) for the myocardium [3]-[5]. This work allowed us to establish a clear relationship between small tissue deformations and speckle pattern motion and demonstrated the potential of speckle pattern tracking as a diagnostic tool to study tissue dynamics.

Meanwhile, Trahey et al. [6] showed experimentally that the motion of speckle patterns produced by blood flow can be used to estimate blood flow velocity. They used a 2-D correlation search algorithm to track the translation of speckle patterns. Our optical flow (velocity field) algorithm approach allows the computation of parameters related to the tissue translation and also to rotation and deformation [3]-[5], Bertrand et al. [7], Chen et al. [8], and Trahey et al. [9] also investigated speckle tracking using a phantom and skeletal muscle [7].

This paper discusses the 2-D analysis of the envelope of returned ultrasound signal over subsequent images to extract soft tissue motions. Based on simulations and a theoretical analysis, the fundamental limitations of speckle tracking to assess soft tissue motion are presented. In particular, it discusses a model to study correlation between speckle pattern motion and tissue motion when a tissue is subjected to a linear geometrical transformation (translation, rotation, and deformation).

II. MODEL OF THE ECHOCOGRAPHIC TEXTURE

Several models to simulate the speckle patterns encountered in typical echographic images were proposed in the literature. Basically, if one assumes linearity and position independent point spread function (PSF) in the far field, the 3-D RF echographic signal I(x, y, z) can be described by a 3-D convolution product (⊗) between the system PSF H(x, y, z) and the impulse response of the tissue T(x, y, z) [2], [11], [12]

\[
I(x, y, z) = H(x, y, z) \otimes T(x, y, z)
\]

(1)

\[
I(x, y, z) = \iiint T(\mu, \nu, \omega) \times H(x - \mu, y - \nu, z - \omega) d\mu \, d\nu \, d\omega
\]

(2)

where x is the direction in which the beam propagates, y the lateral direction in the imaging plane, and z the direction perpendicular to the imaging plane, i.e., the direction of elevation. If one assumes a separable PSF (at least for the z component: \(H(x, y, z) = H(x, y)H(z(z))\)) and a slice through

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\( I(x, y, z) \), say at \( z = 0 \), to simulate the RF image produced during the B-mode scanning, one gets
\[
I(x, y) = H(x, y) \otimes N(x, y) \tag{3}
\]
with
\[
N(x, y) = \int T(x, y, z)H_z(-z) \, dz. \tag{4}
\]
This last pair of equations forms the basis of our model along with an envelope detection procedure to produce the resulting B-scan image \( I_B(x, y) \).

We chose a cosine modulated by a 3-D Gaussian envelope to model a symmetric planar and separable PSF (approximation of a far field PSF)
\[
H(x, y, z) = e^{-\frac{1}{2} \left( \frac{x^2}{\Delta x^2} + \frac{y^2}{\Delta y^2} + \frac{z^2}{\Delta z^2} \right)} \cdot \cos(2\pi f x). \tag{5}
\]
The tissue is assumed to be a collection of inhomogeneities (cells or others) that behave as scatterers. There are several models [13] to define the \( T(x, y, z) \) term; to simplify the tissue model, we assume a large number of very small inhomogeneities with respect to the wavelength of the PSF
\[
T(x, y, z) = \sum_n a_n \delta(x - x_n, y - y_n, z - z_n) \tag{6}
\]
where \( \delta(x, y, z) \) is the 3-D Dirac function or impulse function, \((x_n, y_n, z_n)\) the randomly distributed centers of each inhomogeneity, and \( a_n \) the echogenicity of each scatterer. Considering (4), one obtains
\[
N(x, y) = \int \sum_n a_n \delta(x - x_n, y - y_n, z - z_n)H_z(z) \, dz \tag{7}
\]
\[
N(x, y) = \sum_n a_n H_z(z_n) \delta(x - x_n, y - y_n). \tag{8}
\]
Hence, to model the tissue term in (3), we use a noisy pattern \( N(x, y) \) that will be discussed now.

A. The Process Function \( N(x, y) \)

In practice, the function \( I(x, y) \) is bandlimited by the 2-D PSF, \( H(x, y) \). If we assume that \( H(x, y) \) is narrowband, then for a sufficiently narrow gate function \( \text{rect}(\Delta x, \Delta y) \), (3) could be rewritten as
\[
I(x, y) \approx H(x, y) \otimes \hat{N}(x, y) \tag{9}
\]
with
\[
\hat{N}(x, y) = N(x, y) \otimes \text{rect}(\Delta x, \Delta y) \tag{10}
\]
that is, the spectrum of the \( \text{rect}() \) function is considered to be uniform within the bandwidth of \( I(x, y) \). For a pixel size \((\Delta x, \Delta y)\) at location \((x_0, y_0)\) in the image plane
\[
\hat{N}(x_0, y_0) = \int \int \sum_n a_n H_z(-z_n) \times \delta(x - x_n, y - y_n) \, dx \, dy = \sum_n a_n H_z(-z_n). \tag{11}
\]
That is, if the beam intensity profile \( H_z(z) \) is constant within the thickness of the beam and if cells (scatterers) have equal echogenicity "\( a_n \)" then \( N(x, y) \) is proportional to the number of cells "\( m \)" overlaying the pixel area within the beam thickness. This number of cells at pixel \((x_0, y_0)\) can be modeled as a random variable with a Poisson distribution, and if it averages say more than five or so, the discrete distribution will be close to a sampled Gaussian; however, with (11), the distribution will be continuous by the fact that cells may have nonuniform echogenicity \( a_n \) and will be set at different positions in a beam that does not have a constant intensity profile in \( z \). In our simulations, the beam thickness and pixel area are large enough to count much more than five cells per pixel, therefore \( N(x, y) \) could be modeled as a normal process.2 Thus, to obtain \( N(x, y) \), one simply needs to generate a 2-D normally distributed random field (image).

B. The B-Scan Image

All those assumptions allow an interesting simplification of the real 3-D problem to a much more easier 2-D problem that incorporates analytically the third dimension. In order to get a typical B-scan image \( I_B(x, y) \), an envelope detection procedure is needed. First, the complex pre-envelope is computed. This is easily done by using the Hilbert transform to get the imaginary part and the original RF image as the real part of the pre-envelope. The envelope is then simply computed as the magnitude of the pre-envelope [14].
\[
I_B(x, y) = |I(x, y) + j \times \text{Hilbert}(I(x, y))|. \tag{12}
\]
Examples of the resulting B-scan images are presented in Figs. 1 and 3. The image size is \( 256 \times 256 \) pixels corresponding to \( 5.12 \times 5.12 \) mm. The instrumentation parameters are given in Table I. For more details on this part of the simulation, the reader is referred to one of the following papers [2], [5].

C. Modeling Tissue Dynamics

We start with a 3-D linear motion described by the following tissue position transformation:
\[
\begin{pmatrix}
x_1 \\
y_1 \\
z_1
\end{pmatrix} = \begin{pmatrix}
T & M \nonumber \\
x_0 & y_0 & z_0
\end{pmatrix}. \tag{13}
\]
This transformation defines the motion of a tissue initial position \((x_0, y_0, z_0)\) to its final position \((x_1, y_1, z_1)\) where \( T \) represents a 3-D tissue translation vector and \( M \) the 3-D deformation and rotation of the tissue. A constraint is added to the model to take into account the incompressibility of muscles [15] and soft tissue in general. This constraint for the 3-D linear deformation studied here is simply \( \text{det}(M) = 1 \).

2In our simulations, the pixel area is \( 20 \mu m \times 20 \mu m \) and the PSF thickness is in the order of \( 2.0 \) mm. For example, a typical cardiac cell (scatterer) is \( 15000 \) to \( 20000 \) \( \mu m^3 \), and therefore one obtains \( 400 \) \( \mu m^3 \times \frac{20000 \mu m^3}{15000 \mu m^3} \approx 53 \) fibers/pixel. This number must be reduced by approximately \( 30\% \) to take into account the extracellular medium, which results in a mean clearly larger than five for the random variable "\( m \)."
III. SIMULATION RESULTS

To simplify as much as possible the tissue motion analysis and simulation, the motion is assumed to take place in the x-y plane (image plane) only. In this particular case, we get the following result after polar decomposition:

\[ M = RD \]

\[ T = \begin{pmatrix} a \\ b \\ 0 \end{pmatrix}, \quad R = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \gamma & \beta \\ 0 & 0 & \beta \end{pmatrix} \]

with \( \det(D) = 1 \).

The first term \( T \) is a vector that represents the axial ("x") component and lateral ("y") component translation of the tissue. \( R \) and \( D \) are matrices describing an \( x-y \) plane rotation and biaxial (that is, a deformation along two perpendicular main axes) deformation of the tissue, respectively. The eigenvectors of \( D \) show the deformation main directions and the eigenvalues are, respectively, \( \alpha \) and \( \beta \). The incompressibility constraint for the 2-D motion studied here is simply \( \det(D) = \alpha \beta = 1 \).

To insert this transformation into the previous section model, we can apply this transformation to the 2-D tissue component \( N(x, y) \) by means of a change of variables \((x', y')\) to obtain the resulting transformed tissue projection \( N(x', y') \) with

\[ \begin{pmatrix} x' \\ y' \\ z \end{pmatrix} = (RD)^{-1} \begin{pmatrix} x - a \\ y - b \\ z \end{pmatrix}. \]

However, in our implementation scheme, we preferred to transform the instrumental term (PSF) instead of the tissue term which, relatively speaking, is the same: If one rotates the tissue clockwise, this is relatively equivalent to rotating the PSF counterclockwise. This procedure is quite interesting for the following reason: The transformation can be done easily since the PSF is known analytically. This also means that periodicity and continuity at the edge can be easily preserved after the transformation since the PSF is confined to the center of the image [5], [16].

A. Translation Simulation

Translation is certainly the most simple tissue motion. In the particular case of a linear scanner, one can easily show that for an "image plane" translation of the tissue, an identical translation of the speckle pattern will occur. In fact, for a tissue translation \((a, b, 0)\), the tissue component becomes \( T(x - a, y - b, z) \) and thus from its image plane projection \( N(x - a, y - b) \), one obtains the observed RF image translation from (3)

\[ I(x - a, y - b) = H(x, y) \otimes N(x - a, y - b). \]  \hspace{1cm} (17)

Note that this translation behavior is at the basis of the speckle tracking algorithm developed by Trahey et al. [6], [9]. The point of interest here is that, for a linear scanner, the translation of the tissue infers an identical translation of the resulting speckle pattern but it is interesting to notice that in a sector scan imaging this would not be the case. Here, the PSF will not be isoplanetic in a Cartesian coordinate system, but rather in polar coordinate \((r, \theta)\). Therefore, an \((x, y)\) translation of a small region of tissue would also involve a rotation transformation with respect to the PSF and, as shown below, this would induce speckle decorrelation.

B. Rotation Simulation

Fig. 1 shows simulated echographic textures following a 0-, 2-, and 10-degree tissue rotation around the image center. By
Fig. 2. Correlation between the tissue rotation and the speckle motion. Simulations for a 5-MHz (a) and 3-MHz (b) transducer with 0.5 mm × 1.0 mm (FWHM) PSF shape and a 5-MHz transducer with a 1.0 mm × 1.5 mm PSF (c) are shown. The error bars represent two standard deviations for 20 simulations. The solid curves are obtained theoretically with (25) and (19) (Section IV). Note the larger error bars in the last figure (with a larger PSF) due to the uncertainty introduced by larger speckles in the images, and therefore a smaller number of them to estimate the correlation.

observing carefully these images (using an animation procedure, a manual assessment, or an appropriate algorithm to track the speckles [3]–[9], [30]), one can measure the underlying tissue rotation for small interframe motion. Fig. 1(d) shows the linear transformation that best fits the optical flow (velocity field) [3]–[5], [7], [30] computed from the two images in Fig. 1(a) and (b). As one can observe, there is a clear rotation component for this 2-degree speckle motion. However, it becomes more and more difficult to determine the tissue motion from the speckle motion as the interframe motion gets larger. One can see, for example, that the speckle pattern in Fig. 1(a) is recognizable in (b) (2-degree rotation) but not in (c) (10-degree rotation), therefore is not trackable from (a) to (c). It is therefore virtually impossible to determine the 10-degree rotation that occurs between images (a) and (c) in Fig. 1, as if the speckle patterns had been "corrupted" by a motion-induced noise. In this paper, the word decorrelation [28], [30] is used to refer to this phenomenon because it is more difficult to correlate the observed texture motion and the underlying tissue motion.
In order to quantify this process, we compute a measure of correlation \( \rho_B^D \) that establishes the correspondence between the tissue rotation and the ultrasonic texture motion. This measure can be defined as the correlation between images \( I_D(x', y') \) and \( I_B^D(x, y) \), respectively the \( \theta \)-rotated reference image and the image produced with a \( \theta \) tissue rotation

\[
\rho_B^D = \frac{\text{Covariance}[I_D(x', y'), I_B^D(x, y)]}{\sqrt{\text{Variance}[I_D(x', y')]} \sqrt{\text{Variance}[I_B^D(x, y)]}}.
\]

(18)

Fig. 2(a) shows the results obtained for the PSF described in Table I. Twenty independent simulations for each angle of rotation have been executed to obtain the confidence interval in Fig. 2. As noted above, a global correlation decrease with rotation is obtained. Furthermore, this decrease is reduced with a lower frequency (Fig. 2(b)) and amplified with a wider PSF (Fig. 2(c)) as will be discussed in more details in Section IV.

C. Deformation Simulation

Fig. 3 illustrates the results of axial expansions of 0, 10, and 30 percent \((\alpha = 1.0, 1.1, 1.3, \text{respectively})\) with the corresponding lateral contraction \((\beta = 1)\). Similarly to rotation, by carefully observing these images, one can measure the underlying tissue deformation for small interframe motion, but as the interframe deformation increases, the task is more difficult due to decorrelation. Fig. 3(d) shows the optical flow computed from the two images in (a) and (b).

Again, one can compute the correlation between tissue deformation and speckle motion from the measure of correlation \( \rho_B^D \) between images \( I_D(x', y') \) and \( I_B^D(x, y) \). Fig. 4 shows the results: a larger decrease in the correlation with higher frequencies and/or larger PSF.

IV. RESULT ANALYSIS

To explain these simulated results, a theoretical study of the correlation between these various tissue linear transformations (LT) and the corresponding ultrasonic speckle motions is performed based on a 2-D extension of the envelope cross-correlation analysis of a narrow-band Gaussian noise [16, 17]. Indeed, the RF images generated with the model are narrow-band Gaussian noise since they are produced with a 2-D narrow-band PSF and a Gaussian noise tissue. With the symmetrical PSF used here, one can relate the correlation \( \rho_B \) to the easier-to-compute RF correlation \( \rho_{LT} \) with a series expansion of a hypergeometric function [5, 17, 18]

\[
\rho_B = \frac{\pi}{4\sqrt{1 - \pi}} \left( \rho_{LT} + \frac{1}{16} \rho_{LT}^4 + \frac{1}{64} \rho_{LT}^6 + \ldots \right)
\]

(19)

where \( \rho_B \) can now be simply calculated from RF images instead of B-mode images in (18).

A. Theoretical Analysis

In this subsection, we will develop the \( \rho_{LT} \) term instead of \( \rho_B \) to simplify the calculation without loss of generality knowing the relationship between the two from (19)

\[
\rho_{LT} = \frac{\text{Covariance}[I(x', y'), I^{LT}(x, y)]}{\sqrt{\text{Variance}[I(x', y')]} \sqrt{\text{Variance}[I^{LT}(x, y)]}}
\]

(20)

where \( I(x', y') \) and \( I^{LT}(x, y) \) represent, respectively, the linear transformation (rotation or deformation) of the reference image and the image produced with a linear transformation of the tissue. Since the mean value (DC value) of RF images is zero, one obtains

\[
\rho_{LT}^D = \frac{\int I(x', y') I^{LT}(x, y) dx dy}{\int [I(x', y')]^2 dx dy \int [I^{LT}(x, y)]^2 dx dy}.
\]

(21)

Using the Parseval identity, one can easily transform this equation in the frequency domain

\[
\rho_{LT}^D = \frac{\int [I(u', v')]^2 I^{LT}(u, v) du dv}{\int [I(u', v')]^2 du dv \int [I^{LT}(u, v)]^2 du dv}.
\]

(22)

with \( * \) the complex conjugate, and \((u', v')\) the linear transformation in the frequency domain produced by the spatial domain linear transformation \((x', y')\). Using (3) in the Fourier space, we now show that the RF image correlation coefficient \( \rho_{LT}^D \) reduces to the correlation coefficient of the corresponding PSF, that is, \( H(x, y) \) and its linear transformation \( H^{LT}(x, y) \), as shown in (23) at the bottom of the next page. \( |N(u', v')|^2 \) is a white Gaussian noise power spectrum equal to the \( N(x, y) \) variance and related to the \( \text{consta}nt \) cell density [19]; thus, one obtains

\[
\rho_{LT}^D = \frac{\int [H(u', v')]^2 H^{LT}(u, v) du dv}{\int [H(u', v')]^2 du dv \int [H^{LT}(u, v)]^2 du dv}.
\]

(23)

An instructive mathematical model of \( H(u, v) \) can thus be obtained using a 2-D Gaussian centered on the PSF spatial frequency \( f \) with standard deviation \( \sigma_u \) and \( \sigma_v \), inversely proportional to the PSF spatial ones, \( \sigma_x \) and \( \sigma_y \), respectively; these being related to the axial and lateral speckle size. For rotation and deformation (assuming a narrow-band PSF and

\[
\rho_{LT}^D = \frac{\int [H(u', v')]^2 H^{LT}(u, v) du dv}{\int [H(u', v')]^2 du dv \int [H^{LT}(u, v)]^2 du dv}.
\]

(24)
small rotation and deformation),
\[
\rho_{t,t}^\theta \approx e^{-\frac{1}{2} \left( \frac{\alpha}{\epsilon} \right)^2 \sin^2 \theta}
\]
(25)
\[
\rho_{t,t}^\alpha \approx \frac{2\sqrt{\alpha \beta}}{\sqrt{(\alpha^2 + 1)(\beta^2 + 1)}} e^{-\frac{1}{2} \left( \frac{\alpha}{\epsilon} \right)^2 \frac{(\alpha^2 - 1)^2}{\alpha^2 + 1}}
\]
(26)

These results can be extended to B-scan images using (19) and agree with the simulated one described above (solid curves in Figs. 2 and 4). Note that, within the framework of this model, rotation and deformation correlations are sensitive to the lateral and axial bandwidth, respectively ($\sigma_\alpha$ and $\sigma_\theta$) and the PSF frequency “f” which appear in the exponential term.

\[
\rho_{t,t}^{LT} = \frac{\iint H(u', v') H^*(u, v) |N(u', v')|^2 du dv}{\sqrt{\iint |H(u', v')|^2 |N(u', v')|^2 du dv} \sqrt{\iint |H(u, v)|^2 |N(u, v)|^2 du dv}}
\]
(23)
The lateral deformation parameter $\beta$ does not appear in the exponential term; therefore, $\rho_{t,t}$ is much less affected by lateral deformation than by axial deformation. The observed $\rho_{t,t}$ sensitivity to rotation is also explained by the $\sin^2 \theta$ term in the exponential. Remember that a weak correlation means that speckle tracking of the tissue motion is more difficult. This indicates that, within the framework of our model, in addition to the obvious small interframe motion, a pulse with a low frequency/bandwidth is desirable for a speckle tracking methodology.

B. Correlation with Lateral Translation (Sector Scan Imaging)

In [9], Trahey et al. investigated correlation with translation using a sector-scan system. In our paper, we investigate an isoplanatic PSF which is invariant with respect to translation. In order to extend our result to a sector scan situation and reproduce its speckle motion for a lateral translation $t_x$, we need to translate the region of interest (ROI) laterally and then rotate it. For small translation, the rotation angle is $\sin^{-1}(t_x/d) \approx t_x/d$, where $d$ is the transducer toROI distance. Given this, (25) can be rewritten in terms of lateral translation (for a sector scan), rather than rotation angle. It is developed as follows:

$$\rho_{t,t} \approx e^{-\frac{1}{2}(\frac{x}{\Delta f})^2(sin^2 \theta)}$$

$$\approx e^{-\frac{1}{2}(\frac{t_x}{\Delta f})^2(sin^2 \theta)}.$$  \hspace{1cm} (27)

This last equation can be further simplified for a rectangular transducer using a diffraction formula. In the far field, or for that matter even in focus, the beam-width is proportional to $1/((fD))$, where $D$ is the transducer lateral dimension; therefore, the lateral bandwidth $\sigma_x$ is proportional to the $(fD)$ product. Substituting this in the above equation, one finds

$$\rho_{t,t} \approx e^{-k(\frac{t_x}{\Delta f})^2(sin^2 \theta)} \approx e^{-k(\frac{t_x}{\Delta f})^2(sin^2 \theta)}.$$ \hspace{1cm} (28)

That is to say, for a given (rectangular) transducer-ROI distance $d$, the correlation is only a function of the translation expressed as a fraction of transducer width. This is indeed one of the conclusions of [9].

C. Graphical Analysis

A very interesting qualitative insight into the computation of $\rho_{t,t}$ is obtained using a graphical representation of (3), an approach somewhat similar to [20]. In Fig. 5, the RF image obtained from the linear transformation (LT) of the tissue (rotation and/or deformation), $I^{LT}(x,y)$, and the linear transformation of the reference RF speckle pattern, $I(x',y')$, are shown as overlapping regions in the frequency domain. One circular area represents the 2-D spectrum of $I^{LT}(x,y)$ and results from a spectral "sampling" of the linear transformed tissue by the unmodified PSF and is therefore centered on the PSF central frequency $\nu_x$ with a radius corresponding to its bandwidth $\Delta f_x$. The 2-D spectrum of $I(x',y')$ is the circular (PSF-sampled) area modified by the linear transformation that affects both the tissue and the PSF since it is now the speckle pattern that is subjected to linear transformation. The relative size of the overlapping region is a good indication of the correlation $\rho_{t,t}$ for a given motion since the spatial correlation is related to the product of one spectrum by the conjugate of the other. The left sketch in Fig. 5 represents rotation and the right one axial/lateral deformation. One can now easily see the decreasing correlation described before as a corresponding relative decrease in the overlapping region and understand how the frequency/bandwidth $(f/\Delta f)$ ratio is a critical factor for the decorrelation induced by rotation and axial deformation. For example, doubling the pulse frequency or halving the bandwidth in Fig. 5 clears out any overlap. A more careful investigation reveals that lateral deformation ($\beta$-related) is much less sensitive to decorrelation than the axial deformation ($\sigma_x$-related). With respect to the transducer frequency, the rotation and the axial deformation of the tissue show a better relationship with their respective speckle motion at lower frequencies while lateral deformation correlation is independent of the pulse frequency. With respect to beam (pulse) size parameters, tissue/speckle correlation decreases with rotation when a wide ultrasonic beam is used while the deformation correlation decreases with the axial duration of the pulse.

These graphical interpretations are in perfect agreement with the theoretical ones.

V. CONCLUSION

This study sets the ground for the development of a tool particularly useful for the assessment of biomechanical soft tissue properties based on speckle tracking. From simulations and a theoretical analysis, the fundamental limitations of speckle tracking to assess soft tissue motion were presented. In particular, we have investigated a model to study correlation between speckle pattern motion and tissue motion when a tissue is subjected to a linear geometrical transformation (translation, rotation, and deformation). Indeed, these theoretical results were successfully applied to the determination of myocardial and skeletal muscle dynamics and were also experimentally validated from a phantom study in earlier research [2]-[9], [30]. Many other groups [21]-[26] are working in related

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1Remember that, under LT, expansion in spatial domain becomes compression in the frequency domain.
fields and the approach is certainly promising. More complex (curved) PSF were investigated recently [27], [28] to study speckle motion artifacts [29] that must be removed in order to reveal the tissue motion. These artifacts are due to phase distortion effects in the bandwidth of the PSF. Tissue functions \( T(x, y, z) \) with different density distributions \( N(x, y) \) using different mixtures of small and large scatterers having their own density distributions would also be an interesting area of research to simulate, for example, normal and ischemic tissue, and to evaluate the significance of specular and resolved scatterer components in tissue motion estimation [26]. Finally, more complex 3-D transformations are under investigation to take into account more realistic nonlinear and out of plane (image) motions.

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