Modeling and analysis of an extrinsic Fabry-Perot interferometer performance using MATLAB

Sanjoy Mandal¹, Tarun Kumar Gangopadhyay², Kamal Dasgupta², Tapas Kumar Basak³, Shyamal Kumar Ghosh³

¹College of Engineering & Management, Kolaghat, KTPP Township, Midnapur(east) West Bengal, India, 721171, Telephone:+913228250394, Fax:+913228250880
²Central Glass and Ceramic Research Institute (CSIR), Kolkata-700 032, India
³Department of Electrical Engineering, Jadavpur University, Kolkata-700 032, India

ABSTRACT

In this paper block diagram representation of optical feedback between two resonator mirrors suffering a phase shift in each round trip during propagation depending on the separation is studied and transfer function model of the loss less Fabry-parot cavity(EFPI) is developed in S-domain. An extrinsic Fabry-Perot interferometer with perfectly parallel surface between the cavity shows efficient interference [1-3]. Block diagram model of the interferometer is determined using the proposed transfer function model of the Fabry-parot cavity. Frequency response analysis was carried out in MATLAB environment of the developed model to explain the behavior of the interferometer. Nyquist stability criterion is employed to analyse the behavior of the interferometer considering the conditions, (i) two surfaces are perfectly parallel in the cavity and (ii) with some tilt angle of the second mirror in the cavity. The results indicates efficient interference in case (i) and restricted interference occurs for case (ii). To explain the case (ii) a new parameter is introduced. An attempt is made to analyse the stability of resonant interference pattern with a slight tilt in one of the mirror. The analysis indicates that reflection from two slightly tilt surface may be used to form a Fabry-Parot cavity with certain restrictions. Modified modes in the cavity is characterized with real positive multiplier('p'). Analysis revealed stable solutions at some discrete values of 'p' and the cavity is very much sensitive to variation of 'p' value.

1. INTRODUCTION

A vibration sensor using Fabry-Perot interferometer (FPI) cavity has been constructed from two parallel, highly reflective surfaces separated by a variable distances [1-3]. This type of simple one dimensional resonator system is known as Fabry-Perot etalon. Fabry-Perot etalon is essentially ultra-narrow line width filters characterised by a series of sharp transmission peaks in wavelength space. These peaks are formed when the phase angle of multiple reflected beams within the cavity result in constructive interference at the etalon's exits surface. This phenomenon can be represented using transfer function modeling. If the cavity size varies in response to an applied measurand the result will produce a sensor and it is possible to comment on operational stability of the optical system using Nyquist plot.

It is sometimes difficult to fabricate two perfectly parallel mirror surfaces inside the cavity. Alternatively the analysis with some tilt angle between two mirror surfaces will provide design constraint during fabrication of the sensor. Thus the analysis may be useful for design of sensors, which employ EFPI. The proposed model is developed from an F-P etalon in a reflective configuration with two reflective surfaces R₁ and R₂ and a separation distance d in air [2], as shown in schematic (Fig 1). The FPI cavity is constructed from two parallel plane mirrors. The first mirror is a coating of 25% reflectivity at outer end of the GRIN lens. The second mirror is a polished steel surface.

Figure 1: Theoretical model of a reflective Fabry-Perot etalon [2,3]
2. MODELLING METHODOLOGY

The FP cavity is the most convenient interferometric configuration as it is simply formed from the space between two, typically parallel, mirror surfaces. Now if a wave is transmitted into the cavity the optical delay for successive reflections gives an additional phase difference which correspond to double passage through the cavity [Vaughan, 4]. The round-trip phase lag $\varphi$ within such a cavity is given by [Saleh and Teich, 5]

$$
\varphi = \frac{2\pi (2nd \cos \theta)}{\lambda}
$$

(1)

where $n$ is the refractive index of the medium between the mirrors, $d$ is the mirror separation, $\theta$ is angle of incidence and $\lambda$ is the propagating wavelength. If the cavity is air-filled ($n=1$, and $\lambda$ can be approximated by its free-space value $\lambda_0$) and the incident illumination is normal ($\theta=0$), Eq. 1 becomes

$$
\varphi = \frac{4\pi d}{\lambda_0}
$$

(2)

Now, the Eq. 1 may be considered as a condition of positive feedback shown in (Fig 2) which requires that the output of the system be fed back in phase with the input. Considering electromagnetic wave propagation of light and the reflected light return to the transmitted point and the phenomenon can be represented as below using block diagram model [5,6]. As the light beam is reflected back, the polarization and phase change occur during reverse propagation. Since phase shift occurs during propagation of the light, the forward path transfer function is represented by phase shift only. If the EFPI has a path length $l$, Eq. 2 becomes

$$
\varphi = \frac{4\pi d}{\lambda_0} = \frac{2\pi l}{\lambda_0}
$$

(3)

where, $l=2d$, an

$$
\lambda_0 = \frac{c}{\nu}
$$

(4)

where, $c$ is velocity of light $=3\times10^8$ m/s and $\nu$ is frequency. Again $\omega = 2\pi \nu$ where, $\omega$ is the frequency in rad/sec. If the distance between two mirrors, $d$ is considered as $55 \mu m$ then from Eq. 3 the relation $\varphi = 1.83 \times 10^{-13} \omega$.

The interference will be taken place between the reference input and output of the Fabry-Perot resonator cavity. Considering 25% of the input is allowed to interfere then the overall block diagram may be represented as below shown in (Fig. 3). Here $j\omega$ is represented by complex variable ‘$s$’. Overall transfer function of the block diagram is given by

$$
T = \frac{0.75G + 0.25}{(1 - G)}
$$

(5)

where, $G = \exp(-1.83 \times 10^{-13})$ i.e. forward path open loop transfer function. Using simple MATLAB program it is shown that interference is possible for all practicable values of separation distance $d$. When the reflecting mirror of the Fabry-
Perot etalon makes some angle as shown in (Fig. 4) the transfer function of the optical system will be completely different.

If $\beta$ is considered as the tilt angle between two reflecting mirrors in the Fabry-Perot sensing cavity as shown in Fig. 7, the corresponding two beams reflected from the mirror at an angle $\sigma$ to each other then $\sigma$ is expressed as [Chen, Grattan et al, 7]:

$$\sigma = 2\beta \left( \frac{d}{f - 1} \right)$$  \hspace{1cm} (6)

where, $f$ and $d$ are the focal length of collimating and launch lens and the distance between the lens and the furthest mirror respectively. From common mode condition and the tolerance of parallelism [7],

$$\beta \left( \frac{\lambda}{8a} \left( \frac{d}{f - 1} \right) \right) = \frac{\lambda}{8ap} \left( \frac{d}{f - 1} \right)$$ \hspace{1cm} (7)

Now,

$$\beta = \frac{\lambda}{8ap} \left( \frac{d}{f - 1} \right)$$  \hspace{1cm} (8)

where, $p$ is any real number greater than unity satisfy common mode condition and the tolerance of parallelism. The factor $'p'$ is introduced to satisfy the inequality of the equation 7 which also characterise the mode. Then the phase lag in the Fabry-Perot cavity is given by

$$\varphi = 2\pi \left( \frac{d + \left\{ \frac{d}{\cos \sigma} \} }{\lambda} \right)$$ \hspace{1cm} (9)

From Eqs. 6 & 8,

$$\sigma = \left[ \frac{\lambda}{4ap} \right] = \frac{\pi c}{2ap\omega}$$ \hspace{1cm} (10)

where, $c$ is speed of light wave, $a$ is diameter of the core and $\omega$ is angular frequency of the light wave. From Eqs. 9 & 10,

$$\varphi = \left[ \frac{\omega d (1 + \cos \sigma)}{c \cos \sigma} \right]$$ \hspace{1cm} (11)

Now expanding $\cos \sigma$ and neglecting higher order term

$$\varphi = \left[ \frac{\omega d (4 - \sigma^2)}{c (2 - \sigma^2)} \right]$$ \hspace{1cm} (12)
From Eqs. 10 & 12 and putting $s=j\omega$, considering analytical continuation, where, $s$ is Laplace operator and $j=\sqrt{-1}$.

Here considering the core diameter of single-mode fiber as $a=5\,\mu m$.

$$\varphi = \left[ \frac{\omega d}{c} \left( 4s^2 p^2 + 22 \times 10^{26} \right) \right]$$

(13)

Now new forward path transfer function will be

$$G' = \text{Exp}\left[ -\frac{sd(4s^2 p^2 + 22 \times 10^{26})}{c(2s^2 p^2 + 22 \times 10^{26})} \right]$$

(14)

$$i.e. G' = \left[ \text{Exp}\left( -\frac{sd}{c} \right) \times \text{Exp}(4s^2 p^2 + 22 \times 10^{26}) \times \text{Exp}\left( \frac{1}{2s^2 p^2 + 22 \times 10^{26}} \right) \right]$$

(15)

By approximating the exponential series up to 1st order

$$G' = \left[ \text{Exp}\left( -\frac{sd}{c} \right) \times \left( 1 + 4s^2 p^2 + 22 \times 10^{26} \right) \times \left( 1 + \frac{1}{2s^2 p^2 + 22 \times 10^{26}} \right) \right]$$

(16)

3. SIMULATION OF THE MODEL AND RESULTS

Theoretical simulation is employed to determine the Nyquist diagram using MATLAB. The Nyquist contour, which is eventually entire right half of the complex plane including imaginary axis and when the contour is mapped into G plane the Nyquist diagram is obtained. The exponential function is evaluated by pade approximation [Ogata, 8].

The Nyquist diagram is shown in (Fig. 5) when the two mirrors of the cavity are perfectly parallel. The locus of the Nyquist characteristic function passes through $(-1+j0)$ and this result indicate that neither the light decays nor it increases. The value of $d$ does not effect stability of the system since the Nyquist characteristic function remain same for all practical values of $d$. Hence the optical interference is stable and analogous to an oscillator.

![Nyquist Diagram](image)

Figure 5: Nyquist diagram of the FPI sensor when the two mirrors are parallel in the cavity

Theoretical simulation is also carried out considering tilting angle $\beta$ of the reflecting surface of FPI. The Nyquist characteristic function for $p=13.68$ is shown in (Fig. 6). It can be shown that Nyquist characteristic function also passes through $(-1+j0)$ for other set of values of $p$, as for example $p=12.1333, 135.3$ etc. In such simulation it is observed that the Nyquist characteristic function passes through $(-1+j0)$ which satisfy the necessary condition for possible interference.
Again in the simulation it is also observed that the Nyquist characteristic function does not pass through (–1+j0) for some values such as \( p = 2, 3 \) etc., which does not satisfy the necessary condition for possible interference. Thus the mathematical modeling with tilt angle for some specific values of \( p \) is confirmed with the possibility of interference.

4. CONCLUSIONS

Simple analysis of Fabry-Perot interferometers assumes a perfectly parallel plate cavity with two mirrors. Low cost practical cavity will always have deviation from the standard analytical model. An attempt is made to analyse the stability of resonant interference pattern with a slight tilt in one of the mirror. Nyquist stability criterion is applied to test the performance of EFPI. The analysis indicates that reflection from two slightly tilt surface may be used to form a FP cavity with certain restrictions. Modified modes in the cavity is characterised with real positive multiplier (as mentioned \( p' \)-parameter). Analysis revealed EFPI stable solutions are available at some discrete values of \( p' \) and the cavitivity is very much sensitive to variation of \( p' \) value. The resonant condition may be ceased because locus of the characteristic function is not passing through (–1+j0). Extra attention is required to permanently fixing the parallel plate for EFPI cavity operation.

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