Performance analysis of trellis coded multi-pulse pulse position modulation for deep space optical communication systems

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ABSTRACT

The deep space optical communication is unguided optical communication without fiber, but is power and bandwidth limited. Technique used for communication in such a system is usually IM/DD and the choice of modulation is pulse position modulation since it is extremely power efficient, but it results in an exponential increase in required bandwidth. To overcome this limitation, multi-pulse PPM is used. This scheme increases the throughput without bandwidth expansion, but penalty is paid in terms of degraded BER performance because the signal sets are no more orthogonal. In this paper, Trellis coded modulation schemes using Ungerboeck’s set partitioning methodology to increase the minimum distance among the symbols with simple convolution encoder followed by a mapper is considered for the improvement in the performance.

Keywords: Pulse position modulation, Multi-pulse pulse position modulation, Trellis coded modulation, Set partitioning, Manhattan distance, Convolution encoder.

1. INTRODUCTION

In application such as deep space optical communication and intersatellite links transmission power is severely limited [1-2]. When pulse position modulation (PPM) technique and its other variants are used in such optical communication system, the use of photon counter as a detector has a possibility of very low energy consumption. There are several key aspects of this modulation that are critical to the deployment of PPM for deep space communications. First, the presence of a pulse in the symbol frame regardless of the transmitted symbol benefits the clock recovery subsystem, whereas an on-off keying (OOK) system may suffer a synchronization loss if a long sequence of zeros is encountered. Unlike the OOK scenario, in PPM [3] one does not require \textit{apriori} knowledge of the signal or background noise levels to implement an optimum PPM receiver. The other key requirement of systems considered for space applications is that the peak laser power level must be large enough to survive huge deep space losses. For this reason, Q-switched lasers are typically considered for such applications. The current technology, however, does not support a scenario where a Q-switched laser can be toggled between “on” and “off” states at a high rate, thus severely limiting the data rate that can be supported using an OOK scheme.

The $M$-ary PPM with large $M$ is more energy efficient than its OOK counterpart for deep space applications. However, to achieve a higher bit rate, PPM requires the exponential increase of transmission bandwidth. Although an optical channel potentially has a broad bandwidth, there is a practical limitation of bandwidth [3] in electrical domain. Thus, the exponential increase of transmission bandwidth is a serious problem of PPM. To overcome this problem, multi-pulse PPM (MP-PPM) has been deployed wherein laser is pulsed in many slots in one frame. The MP-PPM reduces the transmission bandwidth to about a half that of $M$-ary PPM at the same transmission efficiency. Further the capacity and the cutoff rate of MP-PPM is larger than those of $M$-ary PPM for same transmission bandwidth in the practical range of the average number of photons per pulse.
In this paper, impact of Trellis coded PPM (T-PPM) on enhancing the BER performance of the deep space optical communication system has been explored. Performance of the Trellis coded MP-PPM (T-MP-PPM) and the criteria for the optimum assignment of symbol set which is invariant to timing offset has also been investigated. The outline of the paper is as follows. In section 2, system model of M-ary PPM has been described. In section 3, Trellis coded modulation (TCM) has been used for enhancing the performance of the M-ary PPM modulation scheme without bandwidth expansion by set partitioning of (256,1) symbol set and using ½ rate convolution encoder. The sections 4 and 5 deal with the system model of MP-PPM and, usage of TCM to improve the performance of the MP-PPM modulation scheme. In section 6, the comparative study of M-ary PPM and MP-PPM is done and finally the conclusions of the study are given in the section 7.

2. SYSTEM MODEL OF M-ARY PPM

In M-ary PPM, symbol duration ($T_{sym}$) is divided into $M$-slots [4] (refer to Fig.1), each of equal duration, $T_{slot}$ ($=T_{sys}/M$). The pulse is transmitted in one of these $M$ slots to convey $\log_2 M$ bits of information.

![Fig. 1. M-ary PPM signaling scheme.](image)

Under Poisson channel assumption, sufficient statistics which is the number of photons observed in each of the ‘$M$’ sub intervals corresponding to the pulse positions can be easily derived. The receiver makes a decision of choosing the symbol interval with the largest number of counts. In case of symbols with equal count, a random choice is made among these symbols. The error probability for uncoded M-ary PPM is upper bounded by union bound [5] and is given by

$$P_{MPPM}^u \leq \frac{1}{M} \sum_{j=0}^{M-1} \sum_{l \neq j, l < j} \Pr(E_{j,l})$$

(1)

where equally likely PPM symbols are assumed. In the above equation, $\Pr(E_{j,l})$ is the pair-wise error probability of making a decision in favor of the $l$th symbol when $j$th symbol is being transmitted and $P_{MPPM}^u$ is the symbol error rate of the uncoded M-ary PPM. This error event [6] is possible only when the integrated count of the $l$th slot exceeds that of the $j$th slot. The pair wise error probability is

$$\Pr(E_{j,l}) = \Pr(N_j \leq N_l)$$

(2)

Here $N_j$ and $N_l$ are independent Poisson random variables with mean $d_j(\lambda_s + \lambda_n)T_{slot}$ and $d_l\lambda_nT_{slot}$ respectively, $\lambda_s$ is the average number of signal photons per second due to signal impinging on the photodetector and $\lambda_n$ the noise photons due to background light and/or detector dark current and $d_{jl}$ the “distance” between symbols $j$ and $l$ i.e.,

$$d_{jl} = |j-l|$$

(3)

and is defined in terms of the numbers of M-ary PPM slots that exists between the positions (in time) of the $j$th and $l$th symbols. It follows [7] from Eqs. (2) and (3).
\[
\Pr \left[ N_j \leq N_l \right] = Q_1 \left( \sqrt{2d_{jl} \frac{\lambda_n T_{slot}}{n}} , \sqrt{2d_{jl} \left( \lambda_n + \lambda_s \right) T_{slot}} \right)
\]

where, \( Q_1(\alpha,\beta) \) is a Macrum’s Q function \([8]\) and is given by

\[
Q_1(\alpha,\beta) = \exp \left( -\frac{(\alpha^2 + \beta^2)}{2} \sum_{k=0}^{\infty} \left( \frac{\beta}{\alpha} \right)^k I_k (\alpha\beta) \right)
\]

Further, simplification of the above equation can be made by using a Chernoff bound \([9, 10]\) which yields

\[
\Pr \left[ N_j \leq N_l \right] \leq \exp \left[ -d_{jl} \left( \sqrt{\frac{\lambda_n T_{slot}}{n}} + \sqrt{\frac{\lambda_s T_{slot}}{n}} - \sqrt{\frac{\lambda_n T_{slot}}{n}} \right)^2 \right]
\]

After combining Eqs.(1), (2) and (5), one can get

\[
P_{\text{MPPM}}^n \leq \frac{1}{M} \sum_{j=0}^{M-1} \sum_{l \neq j}^{M-1} \exp \left[ -d_{jl} \left( \sqrt{\frac{\lambda_n T_{slot}}{n}} + \sqrt{\frac{\lambda_s T_{slot}}{n}} - \sqrt{\frac{\lambda_n T_{slot}}{n}} \right)^2 \right]
\]

where

\[
\lambda_n T_{slot} = \mu R \ln (M) \quad \text{photons/channel}
\]

where \( \mu \) is the number of signal photons per information nat and \( R \) the code rate (number of information symbol per channel symbol). Equation (6a) is the closed form error probability expression for uncoded \( M \)-ary PPM.

3. PERFORMANCE ANALYSIS OF TRELLIS CODED \( M \)-ary PPM

In general for deep space direct detection applications, large detector areas could potentially collect more photons, thus increasing the number of received photons per information bits. Unfortunately, large detector size implies lower detector bandwidth, which tend to smear the observed pulses over several \( M \)-ary PPM slots and hence results in increased error probability. The TCM \([11]\) can be used to overcome the impact of pulse spreading loss. Moreover by applying TCM, performance can be improved at no penalty in bandwidth expansion, which is quite important for bandwidth constrained channels \([12]\).

3.1 Set partitioning of symbol set

Let us consider \( A_0 = \{0,1,\ldots, M-1\} \) as a set containing the uncoded \( M \)-ary PPM symbols. The distance between a pair of symbols \( j \) and \( l \) selected from \( A_0 \) is simply \( |j-l| \). This set is further divided into sets \( B_0 = \{0,2,4,\ldots\} \) and \( B_1 = \{1,3,5,\ldots\} \). It may be noted that a pair of symbols selected from either one of these sets are at a minimum distance of 2, whereas the symbols selected from \( A_0 \) are at a minimum distance of 1. Next we proceed to subdivide set \( B_0 \) to generate sets \( C_0 = \{0,4,8,\ldots\} \) and \( C_2 = \{2,6,10,\ldots\} \). Similarly, \( B_1 \) can also be sub divided to form \( G_1 = \{1,5,9,\ldots\} \) and \( G_3 = \{3,7,11,\ldots\} \). Finally, one last partitioning of the previous sets to form \( M_1 = \{0,8,16,\ldots\} \), \( D_1 = \{4,12,20,\ldots\} \), \( D_2 = \{2,10,18,\ldots\} \), \( D_3 = \{6,14,22,\ldots\} \), \( D_4 = \{1,9,17,\ldots\} \), \( D_5 = \{5,13,21,\ldots\} \), \( D_6 = \{3,11,19,\ldots\} \), \( D_7 = \{7,15,23,\ldots\} \) was performed. The entire process is depicted in Fig. 2 for \( M=256\).
It may be noted that symbols selected from $D_j$ for any $j$ are at a minimum distance of 8. It appears that one can continue this process in hope of increasing the minimum distance beyond eight. However, the minimum distance between a pair of paths through the Trellis (which ultimately dictates the overall performance) cannot be increased indefinitely with further set partitioning. Also, since interest is in eliminating the impact of pulse spreading, a minimum distance of 4 is sufficient to establish orthogonality among the symbols in the set. That is when the imperfect pulse stretches over 2 to 4 $M$-ary PPM slots, a minimum distance of 4 slots among the symbols selected from $C_j$ (for all $j$) insures that there exists no overlap among the pulses in the set. The alphabet size of the $M$-ary PPM is 256 (8 bits), which allows one to encode 7 data bits per T-PPM symbol. We are using a 7/8 convolutional encoder followed by a mapper [13], as shown in Fig. 3.

![Fig. 2. Set partitioning of (256,1) symbol set](image)

![Fig. 3 Rate 7/8 T-PPM encoder followed by a mapper](image)
Fig. 4. Four state Trellis that uses the set partitioning of Fig. 2 for the generation of rate 7/8 T-PPM signals

The rate 7/8, 4 state Trellis depicted in Fig. 4 uses the set partitioning given in Fig. 2. Given that for any path there are 63 other parallel paths between each pair of nodes. An upper bound on the probability of choosing the wrong symbols within one parallel transition may be obtained by using the union bound, resulting in the following expression

$$P_{\text{MPPM}}^{(7/8)} \approx 63 \Pr(E_{1,j+4})$$  \hspace{1cm} (7)

where \( \Pr(E_{1,j+4}) \) represents the pairwise error probability for a pair of PPM symbols with imperfect pulse shapes that are separated by 4 time slots. The approximation in Eqn. (7) is an asymptotic upper bound for the symbol error probability.

Fig. 5. Symbol error probability versus signal photons for (256,1) coded and uncoded M-ary PPM
4. SYSTEM MODEL OF \((M,p)\) MP-PPM

We next consider the case of MP-PPM as shown in the Fig. 6. The laser is pulsed in \(p\) slots in one frame \([14,15]\) consisting of \(M\) slots. It is represented as \((M, p)\) MP-PPM and it can send, \(Q = M^C_p\) kinds of word patterns in one frame. For example, \((32,2)\) MP-PPM has 496 word patterns. It is considered that a \((M, p)\) MP-PPM word has \(p\) rectangular pulses and each pulse has a 100% duty cycle. When \(L\) bits are transmitted in one frame, \(L\) is selected to satisfy the following condition.

\[
L = \log_2 Q
\]

![Fig. 6. M-ary MP-PPM signaling scheme](image)

In \((32,2)\) MP-PPM, \(2^{8}=256\) word patterns out of 496 word patterns are used for transmitting 8 bits of information, which gives us the flexibility to choose the optimum symbol set. The bit rate \(R\) is given by

\[
R = \frac{\log_2 2^L}{MT_{\text{slot}}} = \frac{L}{MT_{\text{slot}}}
\]

where, \(T_{\text{slot}}\) is the slot duration. Optimum \(2^L\) symbol sets minimizes the symbol error probability from \(Q\) word patterns which can be selected from larger set. The number of word patterns increases by increasing the number of pulses in one frame where \(p \leq M/2\). When \(L\) bits are transmitted in one frame, \(M\)-ary PPM requires \(2^L\) slots in one frame, whereas MP-PPM needs \(M\) slots satisfying Eqn. (8). Thus, the number of slots in one frame of MP-PPM is less than that of \(M\)-ary PPM because \(2^L-M \geq 0\). From Eqn. (9), the slot duration \(T_{\text{Slot}}\) increases by decreasing \(M\) provided, \(R\) is fixed. The transmission bandwidth is approximately measured by the inverse of the slot duration. Therefore, for the same frame duration and the same bit rate, decrease in number of the word patterns allows larger slot duration leading to reduction in the transmission bandwidth.

5. PERFORMANCE ANALYSIS OF TRELLIS CODED MP-PPM

5.1.1 The criteria for determining the optimal symbol set

For finding out an optimum symbol set invariant to timing offset, one has to find the word error probability (WEP) of a word with two continuous pulses and the WEP of the word with two separate pulses with imperfect slot synchronization \([14]\). One should select the words with continuous pulses as assigned symbols to minimize the symbol error probability. So we choose all the continuous pulse symbols as an optimum symbol set and from the rest of the symbols, we choose the symbols having maximum distance with each other to minimize the error probability and hence to improve the performance. For \((32,2)\) MP-PPM, using the above criteria, 256 optimum symbols out of 496 symbols are chosen.

5.1.2 The criteria for determining \((d_{\text{free}})\) uncoded

Under Poisson channel assumption, the upper bound on the error probability for uncoded MP-PPM is found by using the union bound given by Eqn.(1). The only difference lies in the computation of the \(d_{ji}\) the “Manhattan distance” \([14,15]\) between symbols \(j\) and \(l\) which is defined here for the two pulse case. Let us consider the Fig. 7 (a) and 7 (b) where symbols having ‘s’ as the subscript for the start symbols and ‘e’ as the subscript for the end symbols. Let us define
The Manhattan distance is given by

$$d_{jl} = \sqrt{a^2 + b^2}$$  \hspace{1cm} (12)

The Manhattan distance ($d_{jl}$) is computed by mapping the optimum symbol set (as per criteria discussed above) in two-dimensional plane. Each symbol set is assigned the pair $(a, b)$, where $a$ and $b$ are computed as per Eqs. (10) and (11). Combining Eqs. (1) and (2), and simplifying it, using Chernoff bound, we get

$$P^u_{MP-PPM} \leq \frac{1}{M} \sum_{j=0}^{M-1} \sum_{l=0}^{M-1} \exp \left[ - d_{jl} \left( \frac{\sqrt{\lambda_n T_{slot}} + \sqrt{\lambda_x T_{Slot}}}{\sqrt{\lambda_x T_{Slot}}} \right) \right]$$  \hspace{1cm} (13)

where,

$$\lambda_n T_{slot} = \frac{\mu R \ln(M)}{P}$$

photons/channel and $\mu$ is the number of signal photons per information nat and $R$ the code rate (number of information symbol per channel symbol). Equation (13) is the closed form error probability expression for uncoded MP-PPM case.

5.1.3 Criteria for determining ($d_{free}$) coded

Trellis coding is applied to MP-PPM case in the same way as with the $M$-ary PPM. Let us introduce $A_0$ (set of symbols defined in Fig. 2) as a set containing the uncoded MP-PPM symbols. It may be noted that the distance between a pair of symbols $j$ and $l$ selected from $A_0$, is the Manhattan distance as defined in Eqn. (12). This set is further divided into sets $B_0$ and $B_1$. A pair of symbols selected from either one of these sets is at a minimum distance of $2\sqrt{2}$, whereas the symbols selected from $A_0$ are also at a minimum distance of $2\sqrt{2}$. Next proceed
to subdivide set \( B_0 \) to generate sets \( C_0 \) and \( C_2 \). \( B_1 \) can also be subdivided to form \( C_1 \) and \( C_3 \). The entire process is depicted in Fig 2. Symbols selected from \( C_j \) for any \( j \) are at a minimum distance of \( 4\sqrt{2} \) for \( M=32 \) and \( p=2 \).

Let us focus attention to the rate 7/8 CE with mapper for the generation of Trellis coded MP-PPM signals which is depicted in Fig. 3, while the corresponding Trellis diagram are shown in Fig 4. For 7/8 CE case, 4 state Trellis is used and 2 bits produced by the encoder are used to select a set from any of the four possible C-type set: \( C_0 \), \( C_1 \), \( C_2 \) and \( C_3 \). The remaining 6 bits produced by the encoder are used to select a signal from the selected set (note that there are 64 signals in any of the four C-type sets). The Table 1 summarizes the coding gain achieved at the different level of partitions, where the asymptotic coding gain is given by

\[
g_\infty = 10 \log \left( \frac{d_{\text{free}}^{\text{coded}}}{d_{\text{free}}^{\text{uncoded}}} \right)
\]  

Table 1 : Asymptotic coding gains for different signal partitioning

<table>
<thead>
<tr>
<th>No. of signal points in each set</th>
<th>No. of partitioned sets</th>
<th>(d_{\text{free}}^{\text{uncoded}})</th>
<th>(d_{\text{free}}^{\text{coded}})</th>
<th>Asymptotic coding gain (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>128</td>
<td>2</td>
<td>2\sqrt{2}</td>
<td>2\sqrt{2}</td>
<td>0</td>
</tr>
<tr>
<td>64</td>
<td>4</td>
<td>2\sqrt{2}</td>
<td>4\sqrt{2}</td>
<td>3.0</td>
</tr>
<tr>
<td>32</td>
<td>8</td>
<td>2\sqrt{2}</td>
<td>\sqrt{40}</td>
<td>3.5</td>
</tr>
<tr>
<td>16</td>
<td>16</td>
<td>2\sqrt{2}</td>
<td>\sqrt{104}</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Fig. 8. Symbol error probability versus number of signal photons for (32,2) coded and uncoded MP-PPM
6. COMPARATIVE ANALYSIS

A comparative study of the coded and uncoded \(M\)-ary PPM and MP-PPM is made below. Here we have given the reduced \(\lambda_s\) requirements for the coded case with respect to the uncoded case in Tables 2 and 3 in the presence of \(\lambda_n = 2\) and \(\lambda_n = 8\). There is also a significant reduction in bandwidth requirement for MP-PPM with respect to the \(M\)-ary PPM case for same BER performance.

Table 2  Performance comparison of \(M\)-ary PPM and MP-PPM for \(\lambda_n=2\)

<table>
<thead>
<tr>
<th>Error rate</th>
<th>((256,1)) (M)-ary PPM</th>
<th>((32,1)) MP-PPM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncoded ((\lambda_n))</td>
<td>Trellis coded ((\lambda_s))</td>
</tr>
<tr>
<td>(10^{-12})</td>
<td>24.8</td>
<td>9.8</td>
</tr>
<tr>
<td>(10^{-9})</td>
<td>19.9</td>
<td>8.5</td>
</tr>
<tr>
<td>(10^{-6})</td>
<td>14.8</td>
<td>6.8</td>
</tr>
<tr>
<td>(10^{-3})</td>
<td>9.2</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table 3  Performance comparison of \(M\)-ary PPM and MP-PPM for \(\lambda_n=8\)

<table>
<thead>
<tr>
<th>Error rate</th>
<th>((256,1)) (M)-ary PPM</th>
<th>((32,1)) MP-PPM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncoded ((\lambda_n))</td>
<td>Trellis coded ((\lambda_s))</td>
</tr>
<tr>
<td>(10^{-12})</td>
<td>35.2</td>
<td>15.4</td>
</tr>
<tr>
<td>(10^{-9})</td>
<td>29.1</td>
<td>13.4</td>
</tr>
<tr>
<td>(10^{-6})</td>
<td>22.3</td>
<td>10.9</td>
</tr>
<tr>
<td>(10^{-3})</td>
<td>14.8</td>
<td>8.2</td>
</tr>
</tbody>
</table>

7. CONCLUSIONS

In this paper we have studied the performance of Trellis coded \(M\)-ary PPM and Trellis coded MP-PPM schemes for deep space optical communication systems. We have observed that, \((32,2)\) uncoded MP-PPM requires lesser number of signal photons than \((256,1)\) uncoded \(M\)-ary PPM. It has been found that Trellis coded modulation helps in further decreasing the required number of signal photons for a given error rate, without any increase in the bandwidth. Moreover the performance of MP-PPM, can be further improved with the proper choice of symbols among different word patterns.

It is seen that for achieving an error rate of \(10^{-6}\) we need 14.8 signal photons in the uncoded case to transmit 8 bits of information, as compared to Trellis coded \(M\)-ary PPM where we need only 6.8 signal photons. For the uncoded MP-PPM we need 11.8 signal photons and with trellis coded MP-PPM, only 8.5 signal photons are required for maintaining the same error rate. In MP-PPM the number of slots in one frame is less than that of \(M\)-ary PPM, that leads to the reduction in transmission bandwidth. For example in \((32,2)\) MP-PPM we need only one eighth of the bandwidth as compared to \(M\)-ary PPM for the transmission of 8 bits of information.

It is interesting to observe that as the background noise photons increases, the gain in signal photons obtained with Trellis coding falls and also there is general trend in the decrease of required number of signal photons with increase in error probability, which implies that coding is more effective for high SNR case.
REFERENCES