TEMPERATURE DEPENDENT SIGNAL-TO-NOISE RATIO AND SENSITIVITY OF A SURFACE PLASMON RESONANCE BASED FIBER OPTIC SENSOR

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ABSTRACT
In the present paper, a theoretical analysis for predicting the effect of temperature on the performance of a fiber optic SPR sensor has been carried out. The sensor configuration is a three layer Kretschmann configuration consisting of fiber core, thin metallic layer, and bulk sensing layer. The optical fiber considered is a multimoded fiber. The effect of temperature is included in terms of thermo-optic effect in fiber core and sensing layer whereas phonon-electron scattering and electron-electron scattering in the metallic layer. The effect of temperature has been studied on the signal-to-noise ratio (SNR) and sensitivity of a fiber optic SPR sensor in different conditions related to sensing layer and metallic layer. The analysis is carried out for both non-remote sensing case as well as remote sensing case. The purpose of the present work is to present the best possible design of a fiber optic SPR sensor against the temperature variation.

Keywords: Surface plasmon resonance, optical fiber, sensor, thermo-optic effect, phonon-electron scattering, electron-electron scattering, sensing layer.

1. INTRODUCTION
Surface plasmon resonance (SPR) has become a powerful tool for sensing purposes since last two decades. In SPR, a TM (transverse magnetic) light wave excites a charge density oscillation (surface plasmon wave, SPW) along the metal-dielectric interface. The phenomenon of SPR has appropriately been used for sensing purposes by a large number of research groups. The Kretschmann-Raether configuration is frequently applied in SPR sensors. In this configuration the metal layer is directly deposited on the base of a coupling prism. The performance of a SPR sensor is evaluated in terms of two parameters: sensitivity and signal-to-noise ratio (SNR). The sensitivity of a SPR sensor is defined as the shift in resonance parameter (either angle or wavelength) per unit shift in refractive index of sensing layer. More the shift in resonance parameter, larger will be the sensitivity. The SNR is an indicator of the accuracy with which the SPR sensor can measure the resonance parameter, and hence the sensing layer index. The SNR is defined as the shift in resonance parameter per unit change in the SPR curve width. The narrower the SPR curve, larger will be the SNR. Due to a complex and fragile structure of prism based SPR sensor, optical fibers have become an efficient alternate to the coupling prism. Optical fibers provide simple and easy sensor design, no electromagnetic interference, and, moreover, the possibility of remote sensing. Among the several physical parameters, the temperature is known to mostly affect the sensor’s performance. In a fiber optic SPR sensor, its variation affects the properties of optical fiber, metal layer, and sensing layer. Therefore, the phenomena of thermo-optic effect (in fiber core, and sensing layer), and phonon-electron scattering along with electron-electron scattering (in metal layer) must also be included to completely study the fiber optic SPR sensor.

In the present work, we have theoretically analyzed the effect of temperature on the performance of a fiber optic SPR sensor. For both non-remote sensing and remote sensing cases, we have studied the effect of temperature on the SNR and sensitivity of a fiber optic SPR sensor to present the best possible design of a fiber optic SPR sensor against the temperature variation.

1. THEORY
The attenuated total reflection (ATR) method with Kretschmann configuration is considered. A small unclad portion of a multimode optical fiber is coated with a metal film (silver or gold). This is further surrounded by the sensing medium. The light is launched into one of the ends of the fiber through a broadband light source by using the proper optics. The other end is connected to a detection system to
record the output signal. The constituents of fiber optic SPR sensor along with their main properties and effect of temperature are as follows:

2.1 Fiber Core
We consider the central core of the optical fiber made of fused silica. The wavelength dependent refractive index of fused silica is expressed according to following dispersion relation:

\[ n(\lambda) = C_0 + C_1 \lambda^2 + C_2 \lambda^4 + \frac{C_3}{\left(\lambda^2 - 1\right)^3} + \frac{C_4}{\left(\lambda^2 - 1\right)^2} + \frac{C_5}{\left(\lambda^2 - 1\right)} \]  

(1)

where the coefficients \( C_0, C_1, C_2, C_3, C_4, C_5 \) and \( l \) have certain numeric values and \( \lambda \) denotes the wavelength (in \( \mu m \)). Eq. (1) is valid for the wavelength range lying between 0.50 \( \mu m \) and 1.6 \( \mu m \). The thermo-optic effect is taken into account for fiber core.

2.2 Metallic Layer
The frequency dependent dielectric function of any metal can be appropriately represented by the Drude formula:

\[ \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\omega_c)} \]  

(2)

where \( \omega_c \) and \( \omega_p \) represent the collision and plasma frequencies, respectively. The plasma frequency varies with temperature due to its volumetric effects and is written as:

\[ \omega_p = \omega_{p0} \left[ 1 + \gamma_e (T - T_0) \right]^{1/2} \]  

(3)

where \( \gamma_e \) is the volume expansion coefficient of the metal, \( T_0 \) is the room temperature, and \( \omega_{p0} \) is the plasma frequency at room temperature.

The variation in temperature affects the collision frequency in two ways namely phonon-electron scattering (\( \omega_{cp} \)) and electron-electron scattering (\( \omega_{ce} \)). The combined effect of the two is

\[ \omega_c = \omega_{cp} + \omega_{ce} \]  

(4)

\( \omega_{cp} \) can be modeled by using the Holstein model of phonon-electron scattering:

\[ \omega_{cp}(T) = \omega_0 \left[ \frac{2}{5} + 4 \left( \frac{T}{T_D} \right)^5 \int_0^{T_0/T} \frac{z^4dz}{e^{z} - 1} \right] \]  

(5)

where \( T_D \) is the Debye temperature of the metal. Here, \( \omega_0 \) is a constant to be determined from the static limit of the equation (5) together with the knowledge of the dc conductivity.

\( \omega_{ce} \) is modeled by the model proposed by Lawrence:

\[ \omega_{ce}(T) = \frac{1}{6n} \pi^4 \frac{\Gamma \Delta}{hE_F} \left[ \left( k_BT \right)^2 + \left( \frac{h\omega}{4\pi^2} \right)^2 \right] \]  

(6)

where \( \Gamma \) is a constant giving the average over the Fermi surface of the scattering probability and \( \Delta \) is the fractional unklapp scattering. Since the metallic film may only expand into the normal direction, one has to employ a corrected thermal expansion coefficient (\( \alpha' \)) for the expansion of the film thickness. The corresponding expression is:

\[ \alpha'(T) = \alpha(T) + \gamma_e \frac{\nu}{\nu + 1} (T - T_0) \]  

where \( \nu \) is the Grüneisen parameter.
\[
\alpha' = \alpha \frac{(1 + \mu)}{(1 - \mu)} \tag{7}
\]

where \(\mu\) being the Poisson’s number and \(\alpha\) the linear expansion coefficient of the metal.

2.3 Sensing Layer

For the present study, we have separately considered both positive as well as negative thermo optic coefficients for the sensing layer which accounts for the temperature dependence of the sensing layer index \(n_s\). The negative thermo optic coefficient usually corresponds to optical polymers.

2.4 Transmitted Power

The p-polarized light beam is launched into one of the ends of the fiber and is detected at the other end of the fiber. To obtain the intensity reflection coefficient (\(R_p\)) for p-polarized light beam and hence the transmitted power we have used the matrix method for a multilayer system\(^\text{11}\). Depending on fiber length (according to application required), there can be two cases:

2.4.1 Small Fiber Case

Number of reflections (%ref) that a ray undergoes in the fiber sensing area depends on the ray angle with the normal to the core-cladding interface (%), core diameter (D), and the length of the sensing region (L). The corresponding expression is:

\[
%_{\text{ref}} = \frac{L}{D \tan \theta} \tag{8}
\]

Thus the generalized expression for the normalized transmitted power, \(P_{\text{trans}}\), by taking multiple reflections into account is

\[
P_{\text{trans}} = R_p^{N_{\text{ref}}(\theta)} \tag{9}
\]

It may be noted that the minimum angle of any guided ray inside the fiber can be equal to the critical angle of the fiber given by

\[
%_{\text{cr}} = \sin^{-1} \left( \frac{n_{\text{cl}}}{n_1} \right) \tag{10}
\]

where \(n_{\text{cl}}\) and \(n_1\) are the refractive indices of the cladding and core respectively.

2.4.1 Long Fiber Case:

The differential equation corresponding to fiber mode continuum is given as\(^\text{12, 13}\):

\[
\frac{\partial P}{\partial z} = -A \theta^2 P + \frac{D_0}{\theta} \frac{\partial}{\partial \theta} \left( \theta \frac{\partial P}{\partial \theta} \right) \tag{11}
\]

On the right side of eq. (11), first term corresponds to attenuation while the second term corresponds to coupling of different fiber modes. The solution of this equation containing least loss takes the Gaussian form. Therefore, one should launch the Gaussian light beam from the input face of fiber as

\[
P_i = P_0 \exp \left( -\frac{\theta^2}{\Theta_0^2} \right) \tag{12}
\]

The Gaussian beam varies with length of the fiber as

\[
P(\theta, z) = f(z) \exp \left( -\frac{\theta^2}{\Theta^2(z)} \right) \tag{13}
\]

After a minor mathematical work out, one gets

\[
f(z) = \frac{P_0 \Theta_0^2}{\Theta_0^2 \sinh (\gamma_\infty z) + \Theta_0^2 \cosh (\gamma_\infty z)} \tag{14}
\]

and,

\[
\Theta^2(z) = \Theta_\infty^2 \left[ \frac{\Theta_\infty^2 \tanh (\gamma_\infty z) + \Theta_0^2}{\Theta_\infty^2 + \Theta_0^2 \tanh (\gamma_\infty z)} \right] \tag{15}
\]
where the parameters $\Theta_0$ and $\Theta_\infty$ are related to the initial and steady state FWHM of the Gaussian beam. Also, $\gamma_\infty$ is related to the steady state loss. Finally, the length dependent transmitted power is given by

$$P_{\text{trans}}(z) = \frac{\int_{\theta_0}^{\pi/2} R_p N_{\text{eff}}(\theta) P(\theta, z) d\theta}{\int_{\theta_0}^{\pi/2} P(\theta, z) d\theta}$$  \hspace{1cm} (15)

3. RESULTS

To get the SNR and sensitivity of the fiber optic sensor, we calculate the transmitted power by using the eqn. (9) for different wavelengths and get the resonance wavelength $\lambda_{\text{res}}$ corresponding to sensing layer index ($n_s$). If the refractive index of the sensing layer is altered by $dn_s$, the resonance wavelength is shifted by $d\lambda_{\text{res}}$. The sensitivity is calculated as the change in resonance wavelength per unit change in sensing layer index. Since the detection accuracy of a SPR sensor is assumed to be inversely proportional to the SPR curve width, we define the SNR as the ratio of change in resonance wavelength to half maximum width of SPR curve. Figure 2(a) shows the variation of signal-to-noise ratio with temperature for both positive and negative thermo-optic coefficient of the sensing layer. Figure 2(b) shows the corresponding sensitivity variation. Both SNR as well as sensitivity decrease with temperature due to SPR curve shift and curve broadening. The sensing layers with positive thermo-optic coefficient show a slightly better sensitivity as well as SNR.

Figure 2: The variation of (a) SNR and (b) Sensitivity with temperature for positive and negative $dn/dT$ values of sensing layer. The two different sensing layer indices were taken as 1.333 and 1.338 with metallic layer as silver. The other parameters are taken as NA = 0.2, D = 400 $\mu$m, L = 15 mm, $dn/dT$ for silica fiber core = $1.28 \times 10^{-3}$ per °K.

Figures 3(a) and 3(b), respectively, show the variation of SNR and sensitivity with temperature for two different metallic layers. It is observed that the metallic layer of silver has larger SNR than that of gold. Gold shows more sensitivity than silver.

Figure 3: The variation of (a) SNR and (b) Sensitivity with temperature for two different metallic layers. The two different sensing layer indices were taken as 1.333 and 1.338.
The above studies have also been extended to long length fibers. Figures 4(a) and 4(b) show the effect of temperature on the SNR and sensitivity for three different combinations of fiber length and initial FWHM of the Gaussian input. The effect of temperature on SNR and sensitivity (i.e., a decay with temperature) is almost similar to that in non-remote sensing case. Clearly, the extent of the above variation varies with fiber length as well as the width of the input Gaussian source. In both the figures, the curves corresponding to fiber lengths 2 and 3 are very close to each other. The above trend is observed for longer fiber lengths also. This again happens due to the saturation of modal power distribution at longer lengths of the fiber.

Figure 4: The variation of (a) SNR and (b) Sensitivity with temperature for three different combinations of fiber length (FL) and initial FWHM of the Gaussian input. The two different sensing layer indices were taken as 1.333 and 1.338. The metallic layer is taken as silver.

4. CONCLUSIONS
A sensing layer with positive thermo-optic coefficient shows a slightly better SNR and sensitivity than that with negative coefficient. It is better to take the metallic layer of silver if one requires a high SNR. Gold provides a better sensitivity. The variation of SNR and sensitivity with temperature is almost same for both metallic layers. The SNR and sensitivity of the sensor depends on length of the fiber up to a certain value. If the fiber is taken longer than that value, the SNR and sensitivity do not vary much. The FWHM of the input Gaussian light also affects the SNR and sensitivity of the sensor and must be chosen carefully.

REFERENCES

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