Coupled-mode analysis of two-parallel circular dielectric waveguides with a weak rotatory effect

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ABSTRACT

The evanescent coupling between two-parallel circular dielectric waveguides with a weak rotatory effect is analyzed using a singular perturbation technique. The analysis is based on the vectorial wave formulation. The coupled-mode equations are derived in closed form which satisfies the Maxwell equations and boundary conditions for the composite waveguide system within the first-order perturbation.

Keywords: evanescent coupling, parallel dielectric waveguide, rotatory effect, singular perturbation technique

1. INTRODUCTION

The evanescent coupling between two-parallel circular dielectric waveguides is widely used in the design of optical fiber couplers\textsuperscript{1}. There exist two orthogonally polarized modes with the same propagation constant in an isolated circular dielectric waveguide. These are the $E_z$-cosine mode and the $E_z$-sine mode. When two circular dielectric waveguides are in close proximity, a directional coupling takes place between the modes of two guides with the same polarization\textsuperscript{2,3} and the power is efficiently transferred from one guide to the other after a particular propagation length. When a weak rotatory effect is introduced into an isolated circular dielectric guide, on the other hand, the $E_z$-cosine mode and $E_z$-sine modes are coupled to form the circularly polarized modes\textsuperscript{4}. If two circular waveguides with a weak rotatory effect are in close proximity, we can expect a directional coupling which incorporates the polarization conversion into the conventional power transfer characteristics.

In this paper, the directional coupling between two-parallel circular dielectric waveguides with a weak rotatory effect is investigated using a self-consistent coupled-mode formulation\textsuperscript{5} in vectorial form. In this approach, the original wave equations for the composite waveguide system are decomposed into an equivalent system of coupled wave equations on the basis of wave fields associated with each of isolated guides without the rotatory effect. The coupled wave equations are solved using the singular perturbation technique\textsuperscript{5} by assuming that the interaction between two guides and the rotatory effect in each guide are small perturbations. Then the coupled-mode equations are obtained in closed form, which satisfy the Maxwell equations and the boundary conditions of the composite waveguide system within the first-order perturbation. The solutions to the coupled-mode equations are presented in closed form.

2. FORMULATION

The geometry considered here is shown in Fig. 1. Two identical circular dielectric waveguides 1 and 2 are situated parallel to each other along the $z$ direction. The radius of the core is $d$ and the separation between two core centers is $h$. We assume that the cores and cladding regions contain a great deal of terbium $T_b$ and magnetized in the $z$ direction. Then the relative permittivity distribution of the two-waveguides system is characterized by the following tensor\textsuperscript{4}:

\[
\overline{\varepsilon}(x,y) = \begin{bmatrix} \varepsilon_0(\rho_i) & -i\varepsilon_x & 0 \\ i\varepsilon_x & \varepsilon_0(\rho_i) & 0 \\ 0 & 0 & \varepsilon_0(\rho_i) \end{bmatrix}
\]  

(1)

\[
\varepsilon_0(\rho_i) = \begin{cases} \varepsilon_f & \text{for } \rho_i \leq d \\ \varepsilon_e & \text{for } \rho_i > d \end{cases} (i=1,2)
\]

(2)
where \( \varepsilon_f \) and \( \varepsilon_c \) are relative permittivities of the unperturbed core and cladding regions, respectively, and \( \varepsilon_x \) is the perturbed relative permittivity caused by the doped \( T_b \) and applied magnetic field which usually satisfies \( \varepsilon_x \ll \varepsilon_f, \varepsilon_c \). When the \( i \)-th core is situated in isolation, the \( E_z \) and \( H_z \) fields satisfy the following coupled wave equations:

\[
\left( \nabla_i^2 + k_i^2 \varepsilon_i(\rho_i) + \frac{\partial^2}{\partial z^2} \right) E_z = -\frac{\varepsilon_x}{\varepsilon_i(\rho_i)} \omega \mu_0 \frac{\partial}{\partial z} H_z \tag{3}
\]

\[
\left( \nabla_i^2 + k_i^2 \varepsilon_i(\rho_i) + \frac{\partial^2}{\partial z^2} \right) H_z = \frac{\varepsilon_x^2}{\varepsilon_i(\rho_i)} k_i^2 H_z + \omega \varepsilon_x \varepsilon_\infty \frac{\partial}{\partial z} E_z \tag{4}
\]

where \( k = \sqrt{\varepsilon_f \mu_c} \) and \( \nabla_i^2 \) is the transverse Laplacian in the local polar coordinate system \((\rho_i, \phi_i)\) attached to the centre of the \( i \)-th core. Other field components can be easily derived in terms of \( E_z \) and \( H_z \). To simplify the notation, \( E_z \) and \( H_z \) are denoted by two scalar functions \( \Phi \) and \( \Psi \). Bearing Eqs. (3) and (4) in mind, we decompose the wave functions and wave equations for the coupled waveguide system as follows:

\[
\Phi = \Phi_i(\rho_i, \phi_i, z) + \Phi_i(\rho_i, \phi_i, z) \tag{5}
\]

\[
\Psi = \Psi_i(\rho_i, \phi_i, z) + \Psi_i(\rho_i, \phi_i, z) \tag{6}
\]

\[
\left( \nabla_i^2 + \frac{\partial^2}{\partial z^2} + k_i^2 \varepsilon_i(\rho_i) \right) \Phi_i = -\frac{\varepsilon_x}{\varepsilon_i(\rho_i)} \omega \mu_0 \frac{\partial}{\partial z} \Psi_i - k_i^2 \Delta \varepsilon_i \Phi_i \tag{7}
\]

\[
\left( \nabla_i^2 + \frac{\partial^2}{\partial z^2} + k_i^2 \varepsilon_i(\rho_i) \right) \Psi_i = \frac{\varepsilon_x^2}{\varepsilon_i(\rho_i)} k_i^2 \Psi_i + \omega \varepsilon_x \varepsilon_\infty \frac{\partial}{\partial z} \Phi_i - k_i^2 \Delta \varepsilon_i \Psi_i \tag{8}
\]

\[
\left( \nabla_i^2 + \frac{\partial^2}{\partial z^2} + k_i^2 \varepsilon_i(\rho_i) \right) \Phi_2 = -\frac{\varepsilon_x}{\varepsilon_i(\rho_i)} \omega \mu_0 \frac{\partial}{\partial z} \Psi_2 - k_i^2 \Delta \varepsilon_i \Phi_i \tag{9}
\]

\[
\left( \nabla_i^2 + \frac{\partial^2}{\partial z^2} + k_i^2 \varepsilon_i(\rho_i) \right) \Psi_2 = \frac{\varepsilon_x^2}{\varepsilon_i(\rho_i)} k_i^2 \Psi_2 + \omega \varepsilon_x \varepsilon_\infty \frac{\partial}{\partial z} \Phi_i - k_i^2 \Delta \varepsilon_i \Psi_i \tag{10}
\]

with

\[
\Delta \varepsilon_i = (\varepsilon_f - \varepsilon_c) U(\rho_i) = \Delta \varepsilon U(\rho_i), \quad U(\rho_i) = \begin{cases} 1 & (\rho \leq \rho_i) \\ 0 & (\rho > \rho_i) \end{cases} \tag{11}
\]

The terms appeared in the right hand sides of Eqs.(7)-(10) may be regarded as the perturbation terms due to the presence of a weak anisotropic permittivity \( \varepsilon_x \) and the adjacent waveguide core. Note that \( \Delta \varepsilon_i \) and \( \Delta \varepsilon_2 \) have nonzero values only inside the core regions waveguides 1 and 2 where the guided fields associated with the waveguides 2 and 1 sufficiently decay, respectively. When the separation distance between two cores goes to infinity and the anisotropic permittivity tends to zero in Eqs.(7) to (10), the two-waveguides system could support two independent guided modes described by the wave functions \( (\Phi_1, \Psi_1) \) and \( (\Phi_2, \Psi_2) \) for the isolated waveguides 1 and 2. The set of Eqs.(7)-(10) are solved using a procedure of a singular perturbation technique\(^3\) based on the multiple space-scales. A non-dimensional small parameter \( \delta \) is used to identify that all terms included in the right-hand sides of Eqs.(7)-(11) start from the order of \( \delta \) in magnitude and the multiple space scales; \( z_0 = z, z_1 = \delta z, z_2 = \delta^2 z, \cdots, z_n = \delta^nz, \cdots \) are introduced. The wave functions \( \Phi_i(\rho_i, \phi_i, z) \) and \( \Psi_i(\rho_i, \phi_i, z) \) expanded as follows:

\[
\Phi_i(\rho_i, \phi_i, z) = \sum_{n=0}^{\infty} \delta^n \Phi_i^{(n)}(\rho_i, \phi_i; z_0, z_1, z_2, \cdots) \tag{12}
\]

\[
\Psi_i(\rho_i, \phi_i, z) = \sum_{n=0}^{\infty} \delta^n \Psi_i^{(n)}(\rho_i, \phi_i; z_0, z_1, z_2, \cdots). \tag{13}
\]

Substituting Eqs.(12) and (13) into Eqs.(7)-(10) and making use of the relation of the derivative expansion

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**Fig. 1 Cross section of a two-parallel circular dielectric waveguides with weak rotatory effect.**
\[
\frac{\partial^2}{\partial z^2} + 2\delta \frac{\partial}{\partial z_0} - 2 \frac{\partial}{\partial z_1} + \delta^2 \left( \frac{\partial^2}{\partial z_1^2} + 2 \frac{\partial}{\partial z_1} \right) + \cdots ,
\]
\[
(14)
\]
a set of coupled wave equations for \( \Phi_i^{(\delta)} \) and \( \Psi_i^{(\delta)} \) is obtained in the respective order \( \delta^n \) of perturbation. These coupled wave equations are solved using a singular perturbation scheme so that the original wave functions given by Eqs.(5) and (6) satisfy the boundary conditions on two core boundaries at \( \rho_1 = d \) and \( \rho_2 = d \).

### 2.1 \( \delta^\delta \) - order problem

The wave equations and the boundary conditions are decoupled. The problem is reduced to a single waveguide problem for each of isolated waveguides 1 and 2. Assuming that the fundamental \( HE_1 \) mode is supported on the waveguides 1 and 2 in isolation, the solutions to \( \Phi_i^{(0)} \) and \( \Psi_i^{(0)} \) are obtained as follows:

\[
\Phi_1^{(0)} = [A_i(z_i) \cos \phi_i + B_i(z_i) \sin \phi_i] R(\rho_i) e^{i\beta z_0}
\]
\[
(15)
\]
\[
\Psi_i^{(0)} = Q(\beta)[A_i(z_i) \sin \phi_i - B_i(z_i) \cos \phi_i] R(\rho_i) e^{i\beta z_0}
\]
\[
(16)
\]

with

\[
R(\rho_i) = \begin{cases} 
\frac{J_i(\alpha \rho_i)}{\alpha \rho_i} & (\rho_i < d) \\
\frac{K_i(\beta \rho_i)}{\beta \rho_i} & (\rho_i > d)
\end{cases}
\]
\[
(17)
\]
\[
Q(\beta) = \frac{1}{\omega \mu_0 W} \left( \frac{1}{u^2} + \frac{1}{w^2} \right)
\]
\[
(18)
\]
\[
W = \frac{J_i(u)}{u J_i(u)} + \frac{K_i(w)}{w K_i(w)}
\]
\[
(19)
\]
\[
a = \sqrt{k^2 \varepsilon_i - \beta^2}, \quad \gamma = \sqrt{\beta^2 - k^2 \varepsilon_i}, \quad u = \alpha d, \quad w = \gamma d
\]
\[
(20)
\]

where \( J_i(u) \) and \( K_i(w) \) are the Bessel function and the modified Bessel function of second kind of the first order, and \( A_i(z_i) \) and \( B_i(z_i) \) are the modal amplitudes of the \( E_z \) -cosine and \( E_z \) -sine modes in the \( i \)-th waveguide. The dependences of \( A_i \) and \( B_i \) on the slow space scale \( z_i \) are determined from the first-order analysis. The propagation constant \( \beta \) satisfies the following dispersion equation:

\[
\frac{\beta^2}{k^2 \varepsilon_i} \left( \frac{1}{u} + \frac{1}{w} \right)^3 = W \left[ \frac{\Delta \varepsilon}{\varepsilon_i} + \frac{J_i'(u)}{u J_i(u)} \right]
\]
\[
(21)
\]

### 2.2 \( \delta^1 \) - order problem

From Eqs.(7) and (8) we have the first-order wave equations for \( \Phi_i^{(1)} \) and \( \Psi_i^{(1)} \) as follows:

\[
\left\{ \begin{align*}
\nabla_i^2 + \frac{\partial^2}{\partial z_0^2} + k^2 \varepsilon_i \right. & \left. \frac{\partial}{\partial z_0} \right\} \Phi_i^{(1)} = -2 \frac{\partial^2}{\partial z_0^2} \Phi_i^{(0)} - \frac{\partial}{\partial z_0} \omega \mu_0 \frac{\partial}{\partial z_0} \Psi_i^{(0)} - k^2 \Delta \varepsilon \Phi_i^{(0)} \quad (\rho_i < d)
\end{align*} \right.
\]
\[
(22)
\]
\[
\left\{ \begin{align*}
\nabla_i^2 + \frac{\partial^2}{\partial z_0^2} + k^2 \varepsilon_i \right. & \left. \frac{\partial}{\partial z_0} \right\} \Phi_i^{(1)} = -2 \frac{\partial^2}{\partial z_0^2} \Phi_i^{(0)} - \frac{\partial}{\partial z_0} \omega \mu_0 \frac{\partial}{\partial z_0} \Psi_i^{(0)} \quad (\rho_i > d)
\end{align*} \right.
\]
\[
(23)
\]
\[
\left\{ \begin{align*}
\nabla_i^2 + \frac{\partial^2}{\partial z_0^2} + k^2 \varepsilon_i \right. & \left. \frac{\partial}{\partial z_0} \right\} \Psi_i^{(1)} = -2 \frac{\partial^2}{\partial z_0^2} \Psi_i^{(0)} + i \omega \varepsilon_i \frac{\partial}{\partial z_0} \Phi_i^{(0)} - k^2 \Delta \varepsilon \Psi_i^{(0)} \quad (\rho_i < d)
\end{align*} \right.
\]
\[
(24)
\]
\[
\left\{ \begin{align*}
\nabla_i^2 + \frac{\partial^2}{\partial z_0^2} + k^2 \varepsilon_i \right. & \left. \frac{\partial}{\partial z_0} \right\} \Psi_i^{(1)} = -2 \frac{\partial^2}{\partial z_0^2} \Psi_i^{(0)} + i \omega \varepsilon_i \frac{\partial}{\partial z_0} \Phi_i^{(0)} \quad (\rho_i > d).
\end{align*} \right.
\]
\[
(25)
\]

Equations (22) - (25) are solved after substituting the zero-order solutions (15) and (16) into the right-hand side of the respective equations. The solutions are given as a sum of the particular solutions related to \( \partial A_i / \partial z_1, \partial B_i / \partial z_1, A_i, B_i, A_2, B_2 \) and the solutions to the homogeneous equations. The first-order boundary conditions on the core of
waveguide 1 require that the tangential electric and magnetic fields derived from \((\Phi_1^{(1)}, \Phi_2^{(1)})\) and \((\Psi_1^{(1)}, \Psi_2^{(1)})\) should be continuous across \(\rho_1 = d\). Applying the first-order boundary conditions, the first-order solutions become singular because the homogeneous parts of the solutions have the same forms as Eqs.(15) and (16) for the zero-order problem. Then the solutions to the first-order problem are allowed only when a solvability condition is satisfied. Similar arguments are applied to the first-order wave equations for \((\Phi_2^{(1)}, \Psi_2^{(1)})\) and the first-order boundary conditions on the core of waveguide 2.

### 2.3 Coupled-mode equations

The solvability conditions for the first-order problem yield a set of linear equations that describe the evolutions of the modal amplitudes \(A_1, B_1, A_2,\) and \(B_2\) as follows:

\[
\begin{align*}
\frac{d}{dz}
\begin{bmatrix}
A_1 \\
B_1 \\
A_2 \\
B_2
\end{bmatrix}
&= \begin{bmatrix}
0 & -\kappa_c & 0 & -i\kappa_s \\
\kappa_c & 0 & 0 & -i\kappa_s \\
-ik_s & 0 & 0 & -\kappa_c \\
0 & -ik_s & \kappa_s & 0
\end{bmatrix}
\begin{bmatrix}
A_1 \\
B_1 \\
A_2 \\
B_2
\end{bmatrix}
\end{align*}
\]

with

\[
\kappa_c = \kappa + \Delta \kappa_c, \quad \kappa_s = \kappa + \Delta \kappa_s
\]

\[
\kappa = \frac{2W}{\delta^2 Y K_1'(w)}
\]

\[
\Delta \kappa_c = \frac{\Delta \varepsilon}{\delta^2 Y K_1'(w)} J_1'(u) \left[ K_0(\gamma h) - K_2(\gamma h) \right]
\]

\[
\Delta \kappa_s = \frac{\Delta \varepsilon}{\delta^2 Y K_1'(w)} J_1'(u) \left[ K_0(\gamma h) + K_2(\gamma h) \right]
\]

\[
\kappa_s = \beta^2 d \varepsilon_c \left( \frac{1}{u^2} + \frac{1}{v^2} \right) \left( \frac{1}{u} J_1(u) + \frac{1}{v} J_1(u) \right) + \frac{2}{\beta^2 d^2}
\]

\[
Y = \beta d \left\{ 2W \left[ Z - \frac{X}{\beta^2 d^2} \right] + \frac{\Delta \varepsilon}{\varepsilon_c} J_1'(u) \left[ Z + \frac{W}{u} - \frac{W}{u} J_1'(u) - \frac{2X}{u} \right] \right\}
\]

\[
X = W \beta^2 d^2 \left\{ \frac{J_1'(u)}{u J_1(u)} + \frac{2}{w^2} \right\} - 1
\]

\[
Z = \frac{K_1'(w)}{wK_1'(w)} \left( W + \frac{1}{u^2} + \frac{1}{w^2} \right)
\]

where the slow space scale \(z_s\) is transformed back into the original space scale \(z\) by letting \(\delta = 1\). In Eq.(26), \(\kappa_c\) and \(\kappa_s\) are the coupling coefficients for the \(E_c\)-cosine and \(E_s\)-sine modes between two-coupled waveguides 1 and 2 without the rotatory effect, and \(\kappa_c\) is the coupling coefficient between \(E_c\)-cosine and \(E_s\)-sine modes in each of the isolated waveguides 1 and 2 under the presence of the weak rotatory permittivity \(\varepsilon_c\). The coupling coefficients \(\kappa_c\) and \(\kappa_s\) consist of a polarization independent coupling coefficient \(\kappa\) and small corrections \(\Delta \kappa_c\) and \(\Delta \kappa_s\), which explain the polarization effect. It is interesting to note that Eq.(26) is similar to the coupled-mode equations derived for a twisted fiber coupler.

The coupled-mode equations (26) can be solved by analyzing the eigenvalues and eigenvectors of the coupling matrix \([\mathbf{K}]\). After tedious but straightforward manipulations, the solutions are obtained as follows:

\[
\begin{bmatrix}
A_1(z) \\
B_1(z) \\
A_2(z) \\
B_2(z)
\end{bmatrix}
= \begin{bmatrix}
A_1(0) \\
B_1(0) \\
A_2(0) \\
B_2(0)
\end{bmatrix}
= \begin{bmatrix}
t_1 & t_2 & t_3 & t_4 & A_1(0) \\
- t_2 & t_3 & - t_4 & t_6 & B_1(0) \\
t_3 & t_4 & t_1 & t_2 & A_2(0) \\
- t_4 & t_6 & - t_2 & t_3 & B_2(0)
\end{bmatrix}
\]
with

\begin{align}
    t_1(z) &= \cos(\sigma z)\cos(\tau z) - \cos(\eta z)\sin(\sigma z)\sin(\tau z) \\
    t_2(z) &= \sin(\eta z)\cos(\sigma z) \\
    t_3(z) &= -i[\cos(\sigma z)\sin(\tau z) + \cos(\eta z)\sin(\sigma z)\cos(\tau z)] \\
    t_4(z) &= -i\sin(\eta z)\sin(\sigma z)\sin(\tau z) \\
    t_5(z) &= \cos(\sigma z)\cos(\tau z) + \cos(\eta z)\sin(\sigma z)\sin(\tau z) \\
    t_6(z) &= -i[\cos(\sigma z)\sin(\tau z) - \cos(\eta z)\sin(\sigma z)\cos(\tau z)]
\end{align}

\[\sigma = \sqrt{k_x^2 + (\Delta k_x - \Delta k_y)^2}, \quad \tau = k + \frac{\Delta k_x + \Delta k_y}{2}\]

\[\cos \eta = \frac{\Delta k_x - \Delta k_y}{2 \sigma}, \quad \sin \eta = \frac{k_x}{\sigma}\]

where \(A_i(0)\) to \(B_i(0)\) denote the modal amplitudes of the initial excitation at \(z = 0\). Let us assume that the waveguide \(I\) is illuminated by a linearly polarized optical wave with a unit power and a polarization angle \(\theta\) relative to the \(y\) axis. Then from Eqs.(35)-(41), the optical power \(P_1(z)\) transmitted through the waveguide \(I\) is obtained as follows:

\[P_1(z) = |A_1(z)|^2 + |B_1(z)|^2 = \frac{1}{2} + \frac{1}{2}M(\eta, \theta, z)\cos[2\tau z + \xi(\eta, \theta, z)] \tag{44}\]

where

\[M(\eta, \theta, z) = 1 - 4\cos^2 \eta \sin^2(\sigma z)[\sin(2\theta)\cos(\sigma z) + \sin \eta \cos(2\theta)\sin(\sigma z)]^2 \tag{45}\]

\[\xi(\eta, \theta, z) = \tan^{-1}\left[\frac{\sin(2\theta)\sin^2(\sigma z) - \cos \eta \cos(2\theta)\sin(2\sigma z)}{\sin^2 \eta + \cos^2 \eta \cos(2\sigma z)}\right] \tag{46}\]

Since the coupled-mode equations (26) fully satisfy the energy conservation relation, the optical power \(P_2(z)\) transferred to the waveguide 2 is given by \(P_2(z) = 1 - P_1(z)\).

3. DISCUSSION

The power transfer function described by Eq.(44) depends on an envelop function \(M(\eta, \theta, z)\), the parameter \(\tau\), and a phase function \(\xi(\eta, \theta, z)\). As \(\varepsilon_x\) and hence \(\kappa\) increase, the modulation depth of the envelop decreases and the power transfer function is mainly governed by the parameter \(\tau\) and function \(\xi(\eta, \theta, z)\). Figure 2 shows a plot of \(P_1(z)\) calculated for the waveguide parameters \(\varepsilon_c = 2.117, (\varepsilon_f - \varepsilon_c)/\varepsilon_f = 0.5, \varepsilon_x = 0.01, h/d = 3.0\), and \(V = kd\sqrt{\varepsilon_f - \varepsilon_c} = 2.0\).
The full solutions (25) depend on the coupling coefficients $\kappa$, $\Delta \kappa_c$, $\Delta \kappa_s$, and $\kappa_x$ in a complicated manner. If the two dielectric waveguides are weakly guiding, the polarization correction terms $\Delta \kappa_c$ and $\Delta \kappa_s$ in the coupling coefficients may be ignored because $\Delta \epsilon / \epsilon_f \ll 1$. Assume that the separation distance $h$ between two core centers and the anisotropic permittivity $\epsilon_x$ are adjusted so that $\Delta \kappa_c \ll \kappa_s$, $\Delta \kappa_s \ll \kappa_s$, and $\kappa_s = \kappa$ are satisfied. When a linearly polarized light of unit power with a polarization angle $\theta$ is launched into the waveguide 1 at $z = 0$ under these circumstances, Eq.(35) is reduced to

\begin{align}
A_1(z) &= \cos(\kappa_x z + \theta) \cos(kz) \\
B_1(z) &= \sin(\kappa_x z + \theta) \cos(kz) \\
A_2(z) &= \cos(\kappa_x z + \theta) \sin(kz) \\
B_2(z) &= \sin(\kappa_x z + \theta) \sin(kz) .
\end{align}

Equations (47)-(50) show that the directional coupling and polarization conversion take place separately in the weakly guiding two-parallel waveguides with a weak rotatory effect. The directional coupling is governed by the coupling coefficients $\kappa$, whereas the polarization conversion angle is by the coupling coefficient $\kappa_x$.

4. CONCLUSION

The coupled-mode equations for two-parallel circular dielectric waveguides with a weak rotatory effect have been derived in closed form using the singular perturbation technique. The equations are self-consistent in the sense that the solutions satisfy the Maxwell equations and boundary conditions for the composite waveguide system within the first-order perturbation. Using the solutions of the coupled-mode equations, the characteristics of the directional coupling and polarization conversion coupling have been discussed.

REFERENCES