A Novel Technique for reducing the imaging domain in microwave imaging of two dimensional circularly symmetric scatterers

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ABSTRACT
A novel strategy for reduction of the imaging domain in microwave imaging of a two dimensional circularly symmetric dielectric scatterer is presented. A custom defined degree of symmetry vector of the measured scattered field, computed as a function of the difference between the first half and the spatially reflected second half of the measured scattered field vector, is employed for the purpose of localizing the scatterer. This reduces the degrees of freedom in the inversion for the unknown object, thereby aiding the global convergence of the solution. The computation time is considerably reduced and the convergence rate is improved. The technique has been tested on synthetic exact and noisy data and the results are promising.

Key words: microwave imaging, 2-D dielectric cylinder, scattering.

1. INTRODUCTION

Inverse profiling with microwaves employ low power microwave radiation to illuminate an object placed in an imaging region. The resulting scattered field measurements at points on a measurement domain outside the imaging region are inverted to get tomographic reconstructions of the object’s complex permittivity. Microwave imaging is of great interest in many applications, ranging from medical imaging to non destructive evaluation of buried pipelines. The measured scattered field and the contrast function of the unknown object whose dielectric profile is to be estimated are nonlinearly related because of multiple scattering [1]. The inverse problem of microwave imaging is ill posed in the Hadamard sense [2]. Generally speaking the ill posedness of the inverse problem is due to the limited amount of data that can be collected.

Several methods to solve this inverse scattering problem have been developed. They could be broadly classified as deterministic and stochastic. All these methods tackle the nonexistence of the inverse scattering problem by redefining it as the minimization of a cost functional. The global minimum of the cost functional gives the final solution. The deterministic algorithms seek to solve the inverse scattering problem by linearizing it around a current estimate and then seeking progressively better estimates of the solution, as examples, the distorted Born iterative method [3], Newton Kantorovich method [4,5] or the modified gradient method [6]. However due to the nonlinearity of the inverse scattering problem, there is a risk that the solution gets trapped in a local minimum. The stochastic methods [7, 8], though globally convergent, are extremely computation intensive, especially when the number of unknowns is high. One way to minimize the risk of local minima is to use prior knowledge about the scatterer, so that the nonlinear problem is linearized about a different background. Such a priori knowledge includes, but is not limited to, the upper and lower bounds of the complex permittivity of the scatterer [4,5] and knowledge of a part of the scatterer [9].

In this paper the imaging of a two dimensional dielectric cylinder is considered, with the assumption that the cross section of the unknown two dimensional object is circular and the complex permittivity distribution symmetric with respect to the centre of the cross section. By a 2-D dielectric cylinder, one refers to the fact that permittivity does not vary along the axis. A circular geometry is employed for the microwave scanner, which helps in generating the degrees of symmetry for the various transmitter positions. It is seen that the degree of symmetry plots exhibit unique features for the direction and distance to the scatterer, from the centre of the imaging domain and this makes it possible to localize the object in the imaging domain. This will reduce the number of degrees of freedom in the inversion for the unknown object, thereby aiding the global convergence of the solution. The computation time is considerably reduced and the convergence rate is improved. The Newton Kantorovich
procedure is employed to image the roughly located object. The details of the formulation of the problem are presented in section 2 and numerical simulations and discussions in section 3.

2. FORMULATION

A 2-D dielectric scatterer of circular cross section and complex permittivity distribution symmetric with respect to the centre of the cross section is located in an imaging domain \( I \), which is usually a square or a rectangle. The maximum diameter possible for the unknown circular object to be imaged is also assumed to be known. This a priori is valid for structures such as dielectric posts and pipe lines. However, since the location of the object to be imaged is not known, the imaging region has to be chosen sufficiently large. The background and the object are non magnetic. To simplify the implementation, TM polarization of the incident field is considered. A circular geometry for the imaging system is employed, with \( M \) line sources equispaced on a circle in the measurement domain. At a given time one of the antennas will be emitting and the others will be receiving. The scattered field is measured for different views. The measurement domain is outside the imaging domain. The circular geometry is selected for the following reasons:

a) Scattered information is collected all around the object for each incidence.

b) It helps in the computation of the degree of symmetry for the different transmitter positions, as will be shown in the following discussion.

The total field satisfies the scalar electric field integral equation for the view \( v \),

\[
e_e(r) = e_{inc}^v(r) + e_{scat}^v(r) \quad \text{…………1},
\]

where \( e_{inc}^v(r) \) is the incident field and \( e_{scat}^v(r) \) is the scattered field given by

\[
e_{scat}^v(r) = \int_I \int k_0^2 c(r') e_e(r') G(r,r') dr' \quad , v = 1,2,\ldots M \quad \text{…………2},
\]

with \( G \) being the two dimensional Greens function given by

\[
G(r,r') = \frac{i}{4} H^{(2)}_0 \left( \frac{k_{ext}}{r-r'} \right) \quad \text{…………3},
\]

c the object contrast given by

\[
c(r) = e_\text{e}(r) - e_\text{ext} \quad \text{…………4},
\]

and \( k_0 \) the free space propagation constant.

The integral equations are discretized with pulse basis functions and point matching. The imaging region is discretized into \( N \) cells. The discretized equations are

\[
[I - G' C] e_v = e_{inc}^v \quad \text{………..5}
\]

and

\[
e_{scat}^v = G_{scat}^v E_v c \quad \text{………..6},
\]

where \( G' \) and \( G_{scat} \) are the integrated Green’s function matrices of dimensions \( N \times N \) and \((M-1) \times N \) where \( M-1 \) is the number of receivers per view, \( C \) and \( E_v \) are diagonal matrices with

\[
[C]_{kk} = c[k] \quad \text{and} \quad [E_v]_{kk} = e_v[k]
\]

where \( c \) and \( e_v \) are the discretized lexicographically ordered contrast function and the total electric field in \( I \) for the view \( v \) respectively.

The degree of symmetry for a transmitter position \( v \) is defined as

\[
s_{real}(v) = \sum_{k=1}^{(M-1)/2} \left\| \text{Re}(e_{scat}^v(k) - e_{scat}^v(M-k)) \right\|^2
\]
\[ s_{\text{imag}}(v) = \sum_{k=1}^{(M-1)/2} \| \text{Im}(e_v^{\text{scat}}(k) - e_v^{\text{scat}}(M-k)) \|^2, \quad v = 1, 2, 3, \ldots M \] 
assumed even number of receivers, and
\[ s_{\text{real}}(v) = \sum_{k=1}^{(M/2-1)} \| \text{Re}(e_v^{\text{scat}}(k) - e_v^{\text{scat}}(M-k)) \|^2 \]
\[ s_{\text{imag}}(v) = \sum_{k=1}^{(M/2-1)} \| \text{Im}(e_v^{\text{scat}}(k) - e_v^{\text{scat}}(M-k)) \|^2, \quad v = 1, 2, 3, \ldots M \] 
assumed odd number of receivers.

When circular geometry of measurement employed, if the dielectric cylinder is circularly symmetric and off centred, the measured scattered field vector \( e_v^{\text{scat}} \) exhibits symmetry with respect to its centre, only for the views \( v_1 \) and \( v_2 \) that are diametrically opposite as shown in figure 1. The symmetry plots also exhibit two maxima, \( a_1 \) and \( a_2 \) which correspond to the transmitter positions where the symmetry of the measured scattered field vector is minimum. The plots of \( s_{\text{real}} \) and \( s_{\text{imag}} \) with respect to \( v \), exhibits two significant minima, at \( v_1 \) and \( v_2 \), and two significant maxima at \( a_1 \) and \( a_2 \) as shown in the example in figure 2. This means that the object centre is located in a line joining the views \( v_1 \) and \( v_2 \). It is clear that \( a_1 \) and \( v_1 \) will be symmetrically located with respect to both \( v_1 \) and \( v_2 \).

When the centre of the circularly symmetric scatterer is much closer to \( v_2 \) than to \( v_1 \), \( a_1 \) and \( a_2 \) will be closer to \( v_2 \) than \( v_1 \). This is explained as follows: the scattered field at a measurement point depends on the distance to the scatterer via the Greens function, and the field inside the object which is a function of the object contrast and the incident field. When the transmitter is at position \( v_1 \) or \( v_2 \), the field inside the object is symmetric about the line joining \( v_1 \) and \( v_2 \). Since the receivers are symmetrically located with respect to the scatterer, the measured scattered field vector will be symmetric about its centre. However when the transmitter position is moved to either side of \( v_2 \) or \( v_1 \), the distances from the adjacent symmetrically located receivers on either side of the transmitter, to the circularly symmetric scatterer are different. This difference is more pronounced when the transmitter position is moved away from \( v_2 \) than when the transmitter position is moved away from \( v_1 \). Also the incident field at the position of the scatterer will exhibit a marked difference in symmetry about the line joining \( v_1 \) and \( v_2 \), the maximum symmetry direction, when the transmitter position is moved away from \( v_2 \) than \( v_1 \). This is because the field is inversely related to the distance. The distance from the transmitter to the position of the scatterer exhibits a larger variation when the transmitter is moved away from position \( v_1 \) than when moved away from position \( v_2 \) along the measurement domain. Therefore the asymmetric points in the symmetry plots will be closer to \( v_2 \) than \( v_1 \), when the scatterer is much closer to \( v_2 \) than \( v_1 \). The values of \( s_{\text{real}} \) and \( s_{\text{imag}} \) at \( a_1 \) and \( a_2 \) will be much larger when the scatterer is farther from the centre of the imaging domain than when it is closer. Also when the scatterer is close to the centre of the imaging domain, the maximum asymmetry positions \( a_1 \) and \( a_2 \) will be almost equidistant from the symmetry positions \( v_1 \) and \( v_2 \). However if the object centre and the centre of the imaging domain coincide, the measured scattered field vector for all the views will be symmetric with respect to its centre and hence the degree of symmetry values for all the views will be very small. Since the maximum diameter possible for the unknown object is assumed to be known, a reduced image may be defined at the centre of the imaging domain in this case. Thus by measuring the maximum values of \( s_{\text{real}} \) and \( s_{\text{imag}} \) and the distance between the maxima, and also noting the direction of symmetry of the scattered field, it is possible to localize the 2-D circularly symmetric dielectric scatterer in the imaging domain. Thus a new reduced imaging domain \( I_r \supset I \) may be defined. This is illustrated in the numerical example in the following section. Since a reduced imaging region is employed, the number of degrees of freedom is reduced and there is a larger data redundancy from which better reconstructions may be obtained. The computation time per iteration for the direct solution of the integral equations are reduced from \( O(N^3) \) to \( O(N_r^3) \) where \( N_r \) is the number of pixels in the reduced imaging domain.
3. NUMERICAL SIMULATIONS AND RESULTS

The coupled equations 1 and 2 are employed to generate synthetic scattered field data in the measurement domain for the inversion, for known object profiles. The degree of symmetry for each transmitter position is computed as per equations 7 or 8, depending on whether even number or odd number of receivers is employed. The variations in the symmetry parameters for varying distances from the centre, permittivities and radii are studied for varying radii and distances of the cylinder centres from the origin, which enable the localization of the cylinder in the imaging domain \( I \), thereby allowing the reduction of the imaging domain to \( I_r \). Some typical symmetry plots are considered in figure 3. The symmetry positions are 19 and 44, where the total number of transceivers employed is 49. The distances between the asymmetry positions through 19 and through 44 along the measurement circle are not as different in figure 3.a, as in figure 3.b. The peak values at the asymmetry positions are much smaller in figure 3.a than in figure 3.b, indicating that the circularly symmetric scatterer is very close to the origin in the first example and farther away from the origin and closer to the transmitter position 44 in the second case. In the case of figure 3.c, the degree of symmetry values for all the transmitter positions are very small, indicating that the object centre and the centre of the imaging domain coincide. The reduced imaging domains in the three cases are chosen accordingly, as indicated in figure 4. The Newton Kantorovich procedure has been employed for the imaging of the approximately located scatterer. The figure 5.a shows the actual profile of a 2-D circularly symmetric dielectric scatterer. Its degree of symmetry values for the different transmitter positions are plotted in figure 5.b. The figure 5.c shows the reconstructed image after the 6th iteration of Newton Kantorovich procedure when the entire imaging region is employed. The reconstructed image when the reduced imaging region is employed is shown in figure 5.d. The result with the proposed technique is seen to be much better. There is also a considerable reduction in computation time compared to the case where the entire imaging domain is employed.

4. CONCLUSION

A novel preconditioning technique for reducing the imaging domain in microwave imaging of two dimensional circularly symmetric dielectric scatterers has been presented. The proposed technique reduces the number of unknowns of the inverse scattering problem of microwave imaging, and also results in considerable reduction in the computation time and improvement in the convergence rate. Since a reduced imaging region is employed, the number of degrees of freedom is reduced and there is a larger data redundancy from which better reconstructions are obtained.

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REFERENCES


Figure 1
Experimental setup. $v_1$ and $v_2$ are the maximum symmetric views, while $a_1$ and $a_2$ are the maximum asymmetric views.

Figure 2
The degree of symmetry values are plotted for various transmitter positions. The minimum values indicate the maximum symmetric positions and the maximum values indicate the maximum asymmetric positions.
Figure 3
a. degree of symmetry plot when the object centre is close to the centre of the imaging domain
b. degree of symmetry plot when the object is nearer to transmitter 44
c. degree of symmetry plot when the centre of the object and the centre of the imaging domain coincide

Figure 4
a. reduced imaging region for the symmetry plots in figure 3.a
b. reduced imaging region for the symmetry plots in figure 3.b
c. reduced imaging region for the symmetry plots in figure 3.c
Figure 5
5 a. The actual profile of a two dimensional circularly symmetric scatterer
5 b. The symmetry plot for the above object profile. The object lies in the line joining the views 19 and 44, and is close to the view 19.
5 c. The reconstructed image after the sixth iteration of the Newton Kantorovich method. The entire imaging domain is employed
5 d. The reconstructed image after the sixth iteration, considering the reduced imaging domain