Lecture Notes
ECE 381

I. Husain

TOPIC 1

Single & Three Phase Circuits

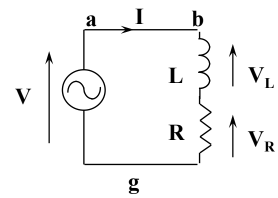
Reference: **Chapter 1**: Zia A.Yamayee and Juan L. Bala Jr. *Electromechanical Energy Devices and Power Systems*, John Wiley & Sons, Inc., 1994.

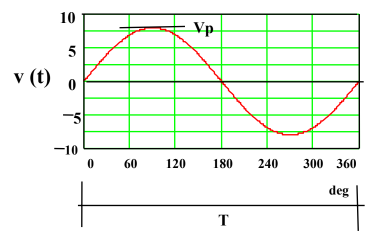
Single-phase Circuits

Single Phase Circuit

Review

- **<u>Single phase circuit components</u>:**
  - Voltage or current sources
  - Impedances (resistance, inductance, and capacitance)
  - The components are connected in series or in parallel.





- The figure shows a simple circuit where a voltage source (generator) supplies a load (resistance and inductance in series).

Single Phase Circuit

Review

- **The voltage source produces a sinusoidal voltage wave**

$$v(t) = \sqrt{2}\, V_{rms} \sin(\omega t)$$

where: V_{rms} is the rms value of the voltage (volts)

ω is the angular frequency of the sinusoidal function (rad/sec)

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{rad/sec} \qquad f = \frac{1}{T} \quad \text{Hz}$$

f is the frequency (60 Hz in USA, 50 Hz in Europe).

T is the time period (seconds).

- **The peak value (max value) of the voltage is** $V_P = \sqrt{2}\, V_{rms}$

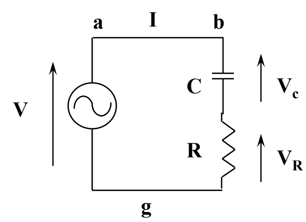
Single Phase Circuit

Review

The rms value is calculated by

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\ dt}$$

- **The voltage direction is indicated by an arrow from g to a. This means during the positive half cycle the potential of point a is larger than g.**



Single Phase Circuit

Review

- The current is also sinusoidal

$$i\,(t) = \sqrt{2}\ I_{rms}\ \sin(\omega t - \phi)$$

where: I_{rms} is the rms value of the current.
ϕ is the phase-shift between current and voltage.

The rms current is calculated by the Ohm's Law:

$$I_{rms} = \frac{V_{rms}}{Z}$$

where: Z is the impedance.

Single Phase Circuit

Review

- The impedances (in Ohms) are :
  - a) Resistance (R)
  - b) Inductive reactance

$$X_L = \omega L$$

  - c) Capacitive reactance

$$X_C = \frac{1}{\omega C}$$

Single Phase Circuit

Review

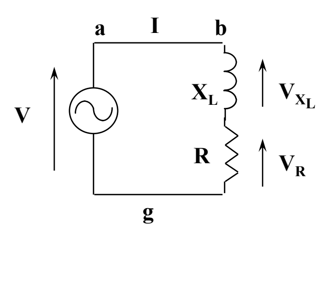
- The impedance of a resistance and a reactance connected in series is :

$$Z = \sqrt{R^2 + X^2}$$

- The phase angle is :

$$\phi = \operatorname{a tan} \frac{X}{R}$$

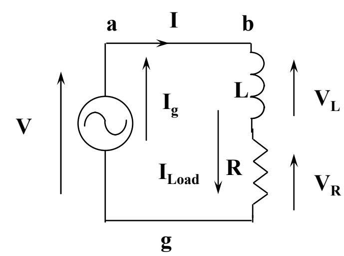
- Impedance calculation



Single Phase Circuit

Review

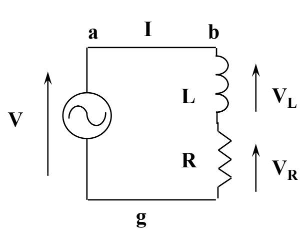
- The generator current flows from **g** to **a** in the positive half cycle.
- **The generator current and voltage are in the same direction.**
- The load current in the positive half cycle flows from **b** to **g** .
- **The load current and voltages are in opposite direction**

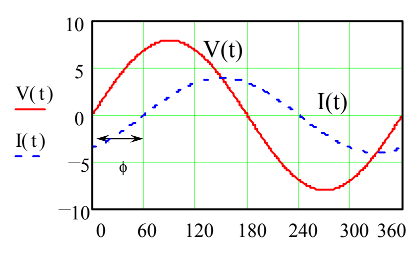


Single Phase Circuit

Review

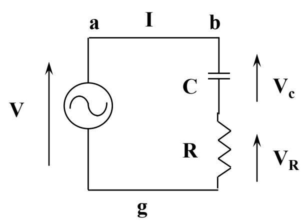
- **Inductive circuit**
  - The ϕ phase-shift between the current and voltage is negative.
  - The current is lagging with respect to the voltage.

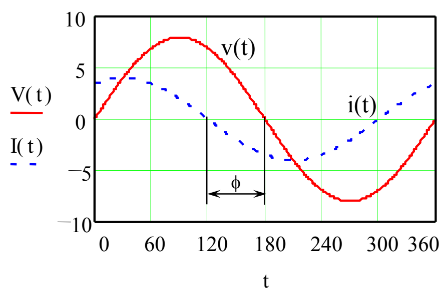




Single Phase Circuit
Review

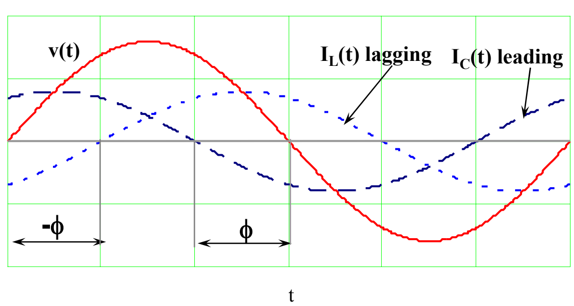
- **Capacitive circuit**
  - The ϕ phase shift between the current and voltage is positive.
  - The current is leading with respect to the voltage.





Single Phase Circuit
Review

- **Illustration of capacitive (leading) and inductive (lagging) current.**



Single Phase Circuit

Review

Complex Notation

- **Engineering calculations need the amplitude (rms value) and phase angle of voltage and current.**
- **The time function is used for transient analysis.**
- **The amplitude and phase angle can be calculated using complex notation.**
- **The voltage, current, and impedance are expressed by complex phasors.**

Single Phase Circuit

Review

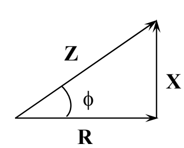
Complex Notation

Impedance phasor: **(resistance, capacitor, and inductance connected in series)**

Rectangular form:

$$\mathbf{Z} = R + j\omega L + \left(\frac{1}{j\omega C}\right) = R + j(X_L - X_C) = R + jX_T$$

Exponential form: $\mathbf{Z} = |Z|\, e^{j\phi}$



where:

$$\mathbf{Z} = \sqrt{R^2 + X^2} \qquad \phi = a\tan\left(\frac{X}{R}\right)$$

Single Phase Circuit

Review

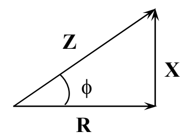
Complex Notation

Impedance phasor: **(resistance, capacitor, and inductance connected in series)**

Polar form:

$$\mathbf{Z} = |Z| \angle\phi = |\mathbf{Z}|[\cos(\phi) + \mathbf{j}\sin(\phi)]$$

$$\mathbf{Z} = \sqrt{R^2 + X^2} \qquad \phi = a\,\mathbf{tan}\,(\frac{X}{R})$$



$$R = \mathbf{Z}\cos(\phi) \qquad X = \mathbf{Z}\sin(\phi)$$

Single Phase Circuit

Review

Complex Notation

- Voltage phasor:

$$\mathbf{V} = |V|\, e^{j\delta} \qquad \text{or}$$

$$\mathbf{V} = |V| \angle\delta = |V|\cos\delta + \mathbf{j}|V|\sin\delta$$

where : V is the rms value, and δ is the phase angle

Note: The supply voltage phase angle is often selected as the reference with δ=0

Single Phase Circuit

Review often

Complex Notation

- Current phasor

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{|V|\, e^{j\delta}}{|Z|\, e^{j\phi}} = \left|\frac{V}{Z}\right| e^{j(\delta-\phi)} = \left|\frac{V}{Z}\right| \left[\cos(\delta-\phi) + j\sin(\delta-\phi)\right]$$

Single Phase Circuit

Review

Kirchhoff 's laws:

- **Voltages:**
  - **The sum of the voltages around any loop is zero.**
- **Other formulation is:**
  - **The sum of generator voltages is equal to the sum of load voltages.**
- **Currents:**
  - **The sum of the currents entering any node point is zero**

Single Phase Circuit

Review

Kirchhoff 's laws:

Example.

– If a generator supplies a resistance, an inductance, and a capacitance connected in series we have:

$$V_g = V_R + V_{X_L} + V_{X_C} = I\,R + I\,j\omega L_{ind} + I\frac{1}{j\,\omega C}$$

– If a generator supplies a resistance, an inductance, and a capacitance connected in parallel we have:

$$I_g = I_R + I_{X_L} + I_{X_C} = \frac{V}{R} + \frac{V}{j\omega L} + \frac{V}{\frac{1}{j\omega C}}$$

Single Phase Circuit

Review

Power calculation.

Instantaneous power is the product of the instantaneous voltage and current.

$$p(t) = v(t)i(t) = \sqrt{2}\ V \sin(\omega t)\sqrt{2}\ I \sin(\omega t - \phi)$$

Where:

$$v(t) = \sqrt{2}\ V \sin(\omega t) \qquad i(t) = \sqrt{2}\ I \sin(\omega t - \phi)$$

Single Phase Circuit

Review

Power calculation. **Instantaneous Power**

Using the sin (α+β) relation we have :

$$p(t) = VI\,\mathbf{cos}(\phi)\left[2\sin^2(\omega t)\right] - VI\,\mathbf{sin}(\phi)\left[2\sin(\omega t)\cos(\omega t)\right]$$

Using the sin^2 (α) and sin (2α) relations the expression for power is:

$$p(t) = VI\cos(\phi)\left[1-\cos(2\omega t)\right] - VI\sin(\phi)\left[\sin(2\omega t)\right]$$

←——— (1) ———→ ←——— (2) ———→

Single Phase Circuit

Review

The power equation is re-arranged as:

$$p(t) = P\left[1-\cos(2\omega t)\right] - Q\left[\sin(2\omega t)\right]$$

←——— (1) ———→ ←——— (2) ———→

Where:

- $P = VI\cos(\phi)$ **is the real power or average power (in watts)**
- $Q = VI\sin(\phi)$ **is the reactive power (in VAR)**

Single Phase Circuit

Review

- <u>Part 1 Real Power</u> $P = VI\cos(\phi)$

The average value of p(t) is the real power. This is the power transferred from the generator to the load.

- <u>Part (2) is the reactive power.</u> $Q = VI\sin(\phi)$

The reactive power average value is zero because it oscillates:

- a) In the positive half cycle the reactive power flows from the generator to the load.

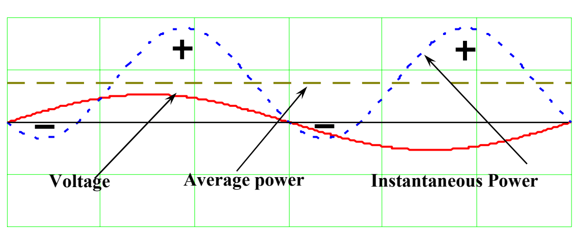
- b) In the negative half cycle the reactive power flows from the load to the generator.

Single Phase Circuit

Review

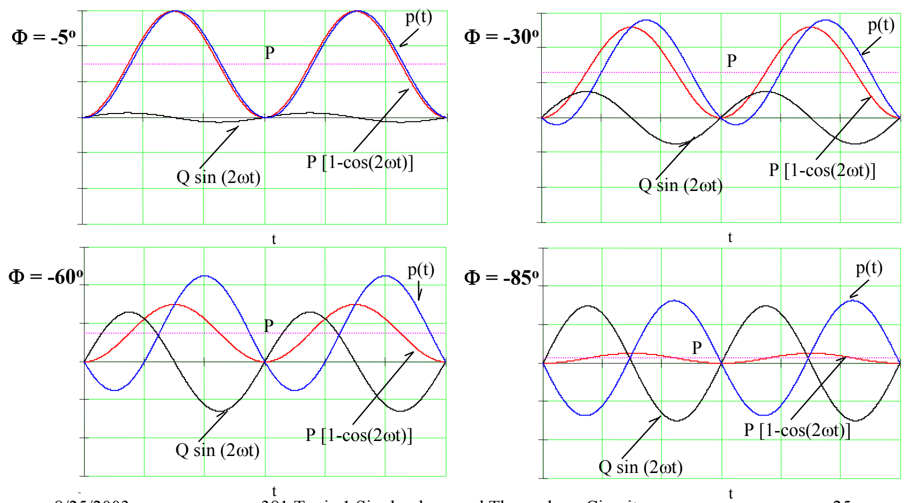
Instantaneous Power Time Function

- Oscillates with double frequency
- Curve shifted, positive area is larger than the negative one.
- The average transmitted power is : $P = \frac{1}{T}\int_0^T p(t)\,dt$



Single Phase Circuit

Reactive and real power waveforms for different phase angle values.



Single Phase Circuit

Review

Complex Power

- **The complex notation can be used for power calculation.**
- **The complex power is defined as : Voltage times the conjugate of the current.**

$$S = V\,\bar{I} = V\;I\;e^{\pm j\phi} = V\,I\left[\cos(\phi) \pm j\sin(\phi)\right] = P \pm jQ$$

- **The power factor magnitude is defined as: the ratio of the real power and the absolute value of the apparent power. The power factor may be lagging or leading.**

$$pf = \cos(\phi) = \frac{P}{|S|} = \cos(\arg(S))$$

Three-phase Circuits

Three-phase Circuits

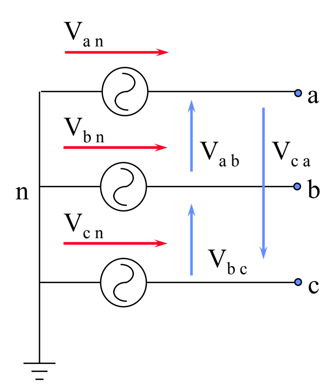
Wye-Connected System

- The neutral point is grounded
- The three-phase voltages have equal magnitude.
- The phase-shift between the voltages is 120 degrees.

$$\mathbf{V_{an}} = |V| \angle 0° = V$$

$$\mathbf{V_{bn}} = |V| \angle -120° = |V|\ \mathbf{e}^{-j120\,\mathbf{deg}}$$

$$\mathbf{V_{cn}} = |V| \angle -240° = |V|\ \mathbf{e}^{-j240\,\mathbf{deg}}$$



Three-phase Circuits

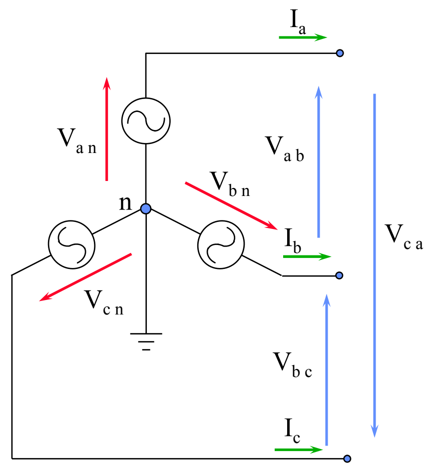
Wye-Connected System

- Line-to-line voltages are the difference of the phase voltages

$$\mathbf{V_{ab}} = \mathbf{V_{an}} - \mathbf{V_{bn}} = \sqrt{3}\ \mathrm{V}\ e^{\,j\,30\,\deg}$$

$$\mathbf{V_{bc}} = \mathbf{V_{bn}} - \mathbf{V_{cn}} = \sqrt{3}\ \mathrm{V}\ e^{-j\,90\,\deg}$$

$$\mathbf{V_{ca}} = \mathbf{V_{cn}} - \mathbf{V_{an}} = \sqrt{3}\ \mathrm{V}\ e^{\,j\,150\,\deg}$$



Three-phase Circuits

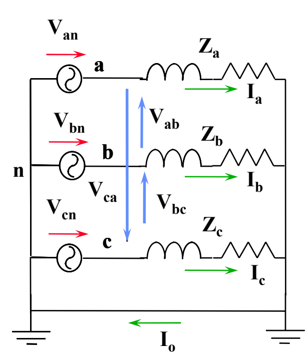
Wye-Connected Loaded System

- **The load is Z_a, Z_b, Z_c**
- **Each phase voltage drives current through the load.**
- **The phase current expressions are:**

$$\mathbf{I_a} = \frac{\mathbf{V_{an}}}{\mathbf{Z_a}} \qquad \mathbf{I_b} = \frac{\mathbf{V_{bn}}}{\mathbf{Z_b}} \qquad \mathbf{I_c} = \frac{\mathbf{V_{cn}}}{\mathbf{Z_c}}$$

- **The system has ground current defined as:**

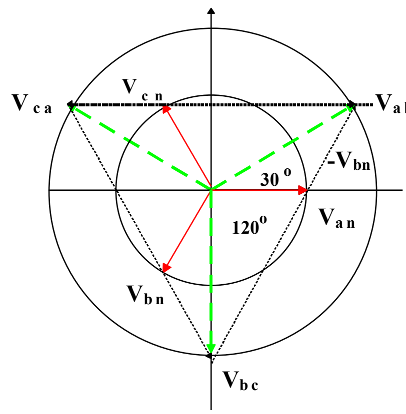
$$\mathbf{I_0} = \mathbf{I_a} + \mathbf{I_b} + \mathbf{I_c}$$



Three-phase Circuits

Wye-Connected System

- Phasor diagram is used to visualize the system voltages
- Wye system has two type of voltages: Line-to-neutral, and line-to-line.
- The line-to-neutral voltages are shifted with 120°
- The line-to-line voltage leads the line to neutral voltage with 30°
- The line-to-line voltage is $\sqrt{3}$ times the line-to-neutral voltage



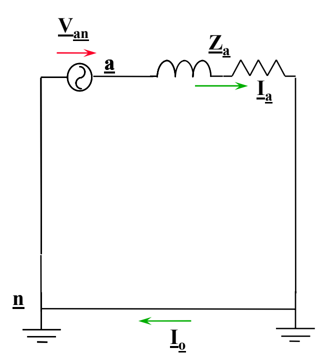
Three-phase Circuits

Wye-Connected Loaded System

- **If the load is balanced ($Z_a = Z_b = Z_c$) then:**

$$I_0 = I_a + I_b + I_c = 0$$

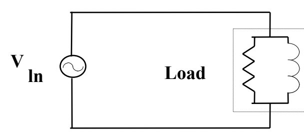
- **This case single phase equivalent circuit can be used (phase a, for instance, only)**
- **Phase b and c are eliminated**



Three-phase Circuits

Wye-Connected System with balanced load

- A single-phase equivalent circuit is used
- Only phase **a** is drawn, because the magnitude of currents and voltages are the same in each phase. Only the phase angles are different (-120^{o} phase shift)
- The supply voltage is the **line to neutral voltage.**
- The single phase loads are connected to neutral or ground.

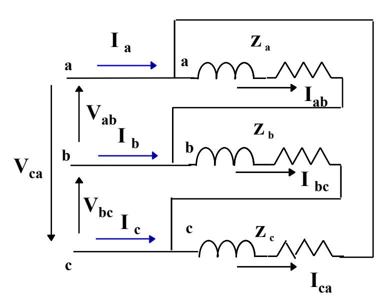


Three-phase Circuits

Balanced Delta-Connected System

- **The system has only one voltage : the line-to-line voltage (V_{LL})**
- **The system has two currents :**
  - **line current**
  - **phase current**
- **The phase currents are:**

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} \qquad I_{bc} = \frac{V_{bc}}{Z_{bc}} \qquad I_{bc} = \frac{V_{bc}}{Z_{bc}}$$



Three-phase Circuits

Delta-Connected System

The line currents are:

$$I_a = I_{ab} - I_{ca}$$

$$I_b = I_{bc} - I_{ab}$$

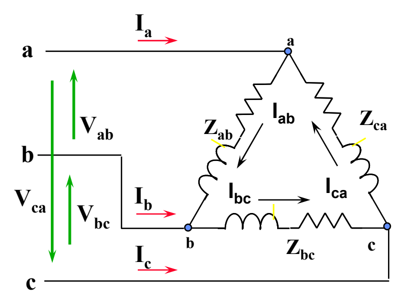
$$I_c = I_{ca} - I_{bc}$$

- **In a balanced case the line currents are:**

$$I_a = \sqrt{3}\, I_{ab}\, e^{-i\,30deg}$$

or

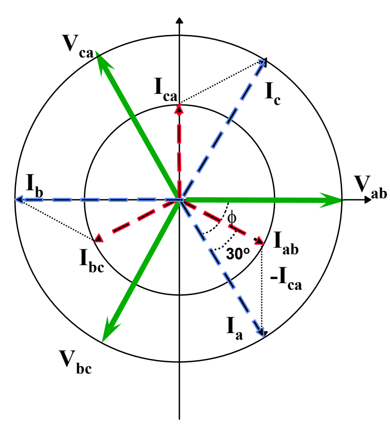
$$I_{line} = \sqrt{3}\, I_{phase}\, e^{-i\,30deg}$$



Three-phase Circuit

Delta-Connected System

- **The phasor diagram is used to visualize the system currents**
- **The system has two type of currents: line and phase currents.**
- **The delta system has only line-to-line voltages, that are shifted by 120°**
- **The phase currents lead the line currents by 30 °**
- **The line current is $\sqrt{3}$ times the phase current and shifted by 30 degree.**



Three-phase Circuit

Power Calculation

- **The three phase power is equal the sum of the phase powers**

$$\mathbf{P = P_a + P_b + P_c}$$

- **If the load is balanced:**

$$\mathbf{P = 3\,P_{phase} = 3\,V_{phase} I_{phase}\cos(\phi)}$$

- **Wye system:** $\mathbf{V_{phase} = V_{LN}}$ $\mathbf{I_{phase} = I_L}$ $\mathbf{V_{LL} = \sqrt{3}\,V_{LN}}$

$$\mathbf{P = 3\,V_{phase} I_{phase}\cos(\phi) = \sqrt{3}\,V_{LL} I_L \cos(\phi)}$$

- **Delta system:** $\mathbf{I_{Line} = \sqrt{3}\,I_{phase}}$ $\mathbf{V_{LL} = V_{phase}}$

$$\mathbf{P = 3\,V_{phase} I_{phase}\cos(\phi) = \sqrt{3}\,V_{LL} I_L \cos(\phi)}$$

Three-phase Circuit

- **Circuit conversions**
  - A delta circuit can be converted to an equivalent wye circuit. The equation for phase a is:

$$\mathbf{Z_a} = \frac{\mathbf{Z_{ab}\,Z_{ca}}}{\mathbf{Z_{ab} + Z_{bc} + Z_{ca}}}$$

  - Conversion equation for a balanced system is:

$$\mathbf{Z_a} = \frac{\mathbf{Z_{ab}}}{3}$$

Three-phase Circuit

<u>**Power measurement**</u>

- **In a four-wire system (3 phases and a neutral) the real power is measured using three single-phase watt-meters.**
- **In a three-wire system (three phases without neutral) the power is measured using only two single-phase watt-meters.**

- The watt-meters are supplied by the line current and the line-to-line voltage.

•- The total power is the algebraic sum of the two watt-meters reading.